IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

# INTUITIONISTIC SUBTRACTIVE FUZZY IDEALS OF A GAMMA SEMIRING

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**Abstract: -** In this paper, we proved that the cartesian product of two intuitionistic subtractive fuzzy ideals of a $\Gamma-$Semiring R is also an intuitionistic subtractive fuzzy ideal. Conversely if $A \times B$ is an intuitionistic subtractive fuzzy left ideal of $R \times R$ then either $A$ or $B$ is an intuitionistic subtractive fuzzy left ideal of $R$

**Keywords:** $\Gamma-$semiring, intuitionistic subtractive fuzzy ideal.

## 1. INTRODUCTION

The set of nonnegative integers N with addition and multiplication gives a characteristics illustrations of a semiring. There are numerous different examples of a semiring, for example, for a given integer n, the set $\{(a_{ij})_{n\times n}\}$ over a semiring structures a semiring with usual addition and multiplication. But the circumstances for the arrangement of the set of the entire negative integers and for the set of all $\{(a_{ij})_{m\times n}\}$ over a semiring R are different. They do not form semirings with the above operations, since the multiplication in the above sense are no longer binary composition. This thought gives another sort of algebraic structure, what is known as a $\Gamma-$ semiring.

The concept of fuzzy set was introduced by L. A. Zadeh [17] in 1965. This pioneering paper of Zadeh was further studied in detail by different researchers. Many mathematicians have applied the concept of fuzzy subsets to the theory of groups, rings, semirings and $\Gamma$ –semirings in algebra [3, 4, 6, 7, 11-12, 14, 16] .The notion of $\Gamma$ in algebra was introduced by N. Nobusawa in 1964 [9]. In 1992, Jun and Lee [6] introduced the notion of fuzzy ideals in a $\Gamma$ –ring. In the year 1995, M.M.K. Rao [10] introduced the concept of $\Gamma$ –semiring R as a generalization of $\Gamma$ – ring. In 1971, Rosenfield [11] defined fuzzy subgroup and fuzzy ideals. In 1986 Atanassov [1] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets.

The motivation for this paper is [2] and the fact that $\Gamma$ –semiring R is a generalization of semiring as well as of $\Gamma$ –ring. The fuzzy concept of Zadeh [16] has been successfully applied to $\Gamma$ – rings and semirings by Jun et.al. [6] and then in $\Gamma$ –semirings by Dutta [3] and Sharma [16]. Here we introduce the concept of product of intuitionistic subtractive fuzzy ideals of R and also some basic properties are derived. Further the relationship between intuitionistic subtractive fuzzy ideals $A$, $B$ and $A \times B$ are proved.

## 2. PRELIMINARIES

In this section, we recall some definitions of $\Gamma$ – semirings, fuzzy ideals and intuitionistic fuzzy ideals which are necessary for this paper.

**Definition 2.1. [16]** Let $S$ *and* $\Gamma$ be non-empty sets. Then S is called a $\Gamma$ – semigroup if there exists a mapping $S \times \Gamma \times S \to S$ denoted by $(x, \alpha, y) \to x\alpha y$ satisfying the condition $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in S$ and *for all* $\alpha, \beta \in \Gamma$.

**Definition 2.2. [16]** Let $R$ and $\Gamma$ be two additive commutative semigroups. Then $R$ is called a $\Gamma -$ semiring if there exists a mapping $R \times \Gamma \times R \to R$ denoted by $x\alpha y$ for all $x, y \in R$ and $\alpha \in \Gamma$ satistifying the following conditions:

**(i)** $x\alpha(y + z) = (x\alpha y) + (x\alpha z)$.

**(ii)** $(y + z)\alpha x = (y\alpha x) + (z\alpha x)$.

**(iii)** $x\ (\alpha + \beta)z = (x\alpha z) + (x\beta z)$.

**(iv)** $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in R$ and $\alpha, \beta \in \Gamma$.

**Example 2.3. (i)** Obviously, every semiring $R$ is a $\Gamma -$ semiring. Let $R$ be a semiring and $\Gamma$ be a commutative semigroup. Define a mapping $R \times \Gamma \times R \to R$ denoted by $x\gamma y = xy$ for all $x, y \in R$, $\gamma \in \Gamma$. Then $R$ is a $\Gamma -$ semiring.

**(ii)** Let $R = (\mathbb{Z}^+, +)$ be the semigroup of non-negative integers and let $\Gamma = (2\mathbb{Z}^+, +)$ be the semigroup of even non negative integers. Then $R$ is a $\Gamma -$semiring.

**Definition 2.4. [16]** A $\Gamma -$ semiring $R$ is said to have a zero element if $0\gamma x = 0 = x\gamma 0$ and $x + 0 = x = 0 + x$ for all $x \in R$ and $\gamma \in \Gamma$.

**Definition 2.5. [16]** A $\Gamma -$ semiring is said to have identity element if there exists $\gamma \in \Gamma$ such that $x\gamma 1 = x = 1\gamma x$ for all $x \in R$.

**Definition 2.6. [16]** A nonempty subset I of a $\Gamma -$ semiring R is called an ideal if $a, b \in I$ implies $a + b \in I$ and $a \in I$, $r \in R$ and $\alpha \in \Gamma$ implies $r\alpha a \in I$ *and* $a\alpha r \in I$.

**Definition 2.7. [2]** A left ideal A of R is called a left subtractive ideal of R if $y, z \in A$, $x \in R$ and $x + y = z$ then $x \in A$. Or

A non- empty ideal I of R is subtractive if and only if $x \in A$, $x + y \in A$ then $y \in A$.

**Definition 2.8. [16]** Let X be a non-empty set. A mapping $\mu : X \to [0,1]$ is called fuzzy subset of X.

**Definition 2.9. [16]** Let $\mu$ be a fuzzy subset of R. Then $\mu$ is called a fuzzy left (right) ideal of R if $\mu(x+y) \geq \mu(x) \wedge \mu(y)$and $\mu(x\alpha y) \geq \mu(y)$ $(\mu(x\alpha y) \geq \mu(x))$, for all $x, y \in R$ and $\alpha \in \Gamma$. A fuzzy ideal of a $\Gamma$ – semiring R is a non-empty fuzzy subset of R which is both fuzzy left and fuzzy right ideal of R.

**Definition 2.10. [2]** Let X be a non-empty set. Then intuitionistic fuzzy set **(IFS)**$A = \{< x, \mu_A(x), \lambda_A(x) >| \, x \in X \text{ and } 0 \leq \mu_A(x) + \lambda_A(x) \leq 1, \text{for all } x \in X\}$, defined on a non-empty set X, where the functions $\mu_A: X \to [0,1]$ and $\lambda_A: X \to [0,1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A respectively.

**Definition 2.11. [2]** Let $R \text{ be a } \Gamma -$ semiring. An **IFS** $A = \{< x, \mu_A(x), \lambda_A(x) >| \, x \in R\}$ is called an intuitionistic fuzzy right (left) ideal of R if it satisfies

(i) $\mu_A(x+y) \geq \mu(x) \wedge \mu(y)$

(ii) $\mu_A(x\alpha y) \geq \mu_A(x)\, (\mu_A(x\alpha y) \geq \mu_A(y))$

(iii) $\lambda_A(x+y) \leq \lambda_A(x) \vee \lambda_A(y)$

(iv) $\lambda_A(x\alpha y) \leq \lambda_A(y)\big(\lambda_A(x+y) \leq \lambda_A(y)\big)$ for all $x, y \in R$ $\alpha \in \Gamma$.

**Definition 2.12. [2]**. An **IFS** $A = (\mu_A, \lambda_A)$ is called an intuitionistic fuzzy ideal of R if it is both intuitionistic fuzzy right and left ideal.

**Definition 2.13.[2]** An **IFS** $A = (\lambda, \mu)$ is called subtractive intuinionistic fuzzy left ideal of R if $y, z \in A, x \in R$ and $x + y = z$ implies that $\mu(x) \geq \mu(y) \wedge \mu(z)$ and $\lambda(x) \leq \lambda(y) \vee \lambda(z)$.

**Definition 2.14. [2]** If $A = (\mu, \lambda)$ be an IFS in R, then strongest intuitionistic fuzzy relation **(SIFR)** on R is $A_R = (\mu_{A_R}, \lambda_{A_R})$, given by $\mu_{A_R}(x,y) = \mu(x) \wedge \mu(y)$and $\lambda_{A_R}(x,y) = \lambda(x) \vee \lambda(y)$ for all $x, y \in R$.

**Definition 2.15.** Let X be a non-empty set. Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be intuitionistic fuzzy sets of X. Then the Cartesian product $A \times B$ is defined as follows;

$$A \times B = \{((x.y), (\mu_A(x) \wedge \mu_B(y)), (\lambda_A(x) \vee \lambda_B(y)) \,|\, x, y \in X\}.$$

## 3. Properties of IFS and SIFR in a $\Gamma -$ semiring

In this section, we review the results with the notion of subtractive fuzzy ideals of a $\Gamma$ −Semiring and characterize the results regarding **IFS** and **SIFR** from [2] in $\Gamma$ −Semirings and then prove that the cartesian product of two intuitionistic subtractive fuzzy ideals of a $\Gamma$ −Semiring R is also an intuitionistic subtractive fuzzy ideal. Conversely if $A \times B$ is an intuitionistic subtractive fuzzy ideal of $R \times R$ then either $A$ or $B$ is an intuitionistic subtractive fuzzy ideal of $R$.

**Remark 3.1.** Let A and B be two IFS of a set X. Then the following results hold:

**(i)** $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\lambda_A(x) \geq \lambda_B(x)$.

**(ii)** $A = B$ if and only $A \subseteq B$ and $B \subseteq A$.

**(iii)** $A^C = \{< x, \mu_A(x), \lambda_A(x) > | \, x \in X\}$.

**(iv)** $A \cap B = \{< x, \mu_A(x) \wedge \mu_B(x), \lambda_A(x) \vee \lambda_B(x) > \; |x \in X\}$.

**(v)** $A \cup B = \{< x, \mu_A(x) \vee \mu_B(x), \lambda_A(x) \wedge \lambda_B(x) > | \, x \in X\}$.

Now the following theorems from [2] which are proved for semirings are also holds for $\Gamma -$ semirings.

**Theorem 3.2.** Let R be a $\Gamma -$ semiring with zero element. Let $A_R$ be the **SIFR** on R, for a given **IFS** A in R. If $A_R$ is an intuitionistic fuzzy left subtractive ideal of $R \times R$ then $\mu_A(x) \leq \mu_A(0)$ and $\lambda_A(x) \geq \lambda_A(0)$ for all $x \in R$.

**Proof.** Let $A_R$ be an intuitionistic subtractive fuzzy left ideal of $R \times R$, then $\mu_{A_R}(x,x) \leq \mu_{A_R}(0,0)$ and $\lambda_{A_R}(x,x) \geq \lambda_{A_R}(0,0)$ for all $x \in R$. This implies that $\mu_A(x) \wedge \mu_A(x) \leq \mu_A(0) \wedge \mu_A(0)$ and $\lambda_A(x) \vee \lambda_A(x) \geq \lambda_A(0) \vee \lambda_A(0)$, which implies that $\mu_A(x) \leq \mu_A(0)$ and $\lambda_A(x) \geq \lambda_A(0)$.

Let X be a non-empty set. If $A$ and $B$ be intuitionistic fuzzy sets of X and $(a,b), \ (c,d) \in A \times B$ then define $(a,b)\alpha(c,d) = (a\alpha c, b\alpha d)$ for all $\alpha \in \Gamma$.

**Theorem 3.3.** Let R be a $\Gamma -$ semiring and $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be intuitionistic subtractive fuzzy left ideals of R. Then $A \times B$ is an intuitionistic subtractive fuzzy left ideals of $R \times R$.

**Proof.** Let $(x_1, x_2), (y_1, y_2) \in R \times R$. Then

$$(\mu_A \times \mu_B)\big((x_1, x_2) + (y_1, y_2)\big) = (\mu_A \times \mu_B)(x_1 + y_1, x_2 + y_2)$$

$$= \mu_A(x_1 + y_1) \wedge \mu_B(x_2 + y_2)$$

$$\geq \{\mu_A(x_1) \wedge \mu_A(y_1)\} \wedge \{\mu_B(x_2) \wedge \mu_B(y_2)\}$$

$$= \{\mu_A(x_1) \wedge \mu_B(x_2)\} \wedge \{\mu_A(y_1) \wedge \mu_B(y_2)\}$$

$$= \{(\mu_A \times \mu_B)(x_1, x_2) \wedge (\mu_A \times \mu_B)(y_1, y_2)\}$$

Similarly,

$$(\lambda_A \times \lambda_B)\big((x_1, x_2) + (y_1, y_2)\big) \leq \{(\lambda_A \times \lambda_B)(x_1, x_2) \vee (\lambda_A \times \lambda_B)(y_1, y_2)\}.$$

Further, for all $\alpha \in \Gamma$,

$$(\mu_A \times \mu_B)\big((x_1, x_2)\alpha(y_1, y_2)\big) = (\mu_A \times \mu_B)(x_1\alpha\, y_1, x_2\alpha\, y_2)$$

$$= \{\mu_A(x_1\alpha\, y_1) \wedge \mu_B(x_2\alpha\, y_2)\}$$

$$\geq \{\mu_A(y_1) \wedge \mu_B(y_2)\}$$

$$= (\mu_A \times \mu_B)(y_1, y_2).$$

Similarly,

$$(\lambda_A \times \lambda_B)((x_1, x_2)\alpha(y_1, y_2)) = (\lambda_A \times \lambda_B)(x_1\alpha\, y_1, x_2\alpha\, y_2)$$

$$= \{\lambda_A(x_1\alpha\, y_1) \vee \lambda_B(x_2\alpha\, y_2)\}$$

$$\leq \{\lambda_A(y_1) \vee \lambda_B(y_2)\}$$

$$= (\lambda_A \times \lambda_B)(y_1, y_2).$$

Hence $A \times B$ is an intuitionistic fuzzy left ideal of $R \times R$. Now let $(a_1, a_2)$, $(b_1, b_2) \in A \times B$ and $(x_1, x_2) \in R \times R$ be such that $(x_1, x_2) + (a_1, a_2) = (b_1, b_2)$ that is $(x_1 + a_1, x_2 + a_2) = (b_1, b_2)$. It follows that $x_1 + a_1 = b_1$ and $x_2 + a_2 = b_2$. Therefore it is clear that

$(\mu_A \times \mu_B)(x_1, x_2) \geq \{(\mu_A \times \mu_B)(a_1, a_2) \wedge (\mu_A \times \mu_B)(b_1, b_2)\}$ and $(\lambda_A \times \lambda_B)(x_1, x_2) \leq \{(\lambda_A \times \lambda_B)(a_1, a_2) \vee (\lambda_A \times \lambda_B)(b_1, b_2)\}$. Hence $A \times B$ is an intuitionistic subtractive fuzzy left ideal of $R \times R$.

Now, we state the following theorem, the proof of which is analogous to the proof of the theorem in [2].

**Theorem 3.4.** Let R be a $\Gamma -$ semiring with zero element and $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be two IFS in R such that $A \times B$ is an intuitionistic subtractive fuzzy left ideals of $R \times R$. Then

**(i)** Either $\mu_A(x) \leq \mu_A(0)$ and $\lambda_A(x) \geq \lambda_A(0)$ or $\mu_B(x) \leq \mu_B(0)$ and $\lambda_B(x) \geq \lambda_B(0)$ for all $x \in R$.

**(ii)** If $\mu_A(x) \leq \mu_A(0)$ and $\lambda_B(x) \geq \lambda_B(0)$ for all $x \in R$, then either $\mu_A(x) \leq \mu_B(0)$, $\lambda_A(x) \geq \lambda_B(0)$ or $\mu_B(x) \leq \mu_B(0)$, $\lambda_B(x) \geq \lambda_B(0)$.

**(iii)** If $\mu_B(x) \leq \mu_B(0)$ and $\lambda_B(x) \geq \lambda_B(0)$ for all $x \in R$, then either $\mu_A(x) \leq \mu_A(0)$, $\lambda_A(x) \geq \lambda_A(0)$ or $\mu_B(x) \leq \mu_A(0)$, $\lambda_B(x) \geq \lambda_A(0)$.

**Proof.** It is routine matter and one can be easily verified.

**Theorem 3.5.** Let R be a $\Gamma -$ semiring with zero element and $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be two IFS in R such that $A \times B$ is an intuitionistic subtractive fuzzy left ideals of $R \times R$. Then

**(i)** If $\mu_B(x) \leq \mu_A(0)$ and $\lambda_B(x) \geq \lambda_A(0)$ for any $x \in R$, then B is an intuitionistic subtractive fuzzy left ideals of R.

**(ii)** If $\mu_A(x) \leq \mu_A(0)$, $\lambda_A(x) \geq \lambda_A(0)$ for all $x \in R$ and $\mu_B(y) > \mu_B(0)$, $\lambda_B(y) \leq \lambda_B(0)$ for some $y \in R$. Then A is an intuitionistic subtractive fuzzy left ideals of R.

**Proof. (i)** If $\mu_B(x) \leq \mu_A(0)$ and $\lambda_B(x) \geq \lambda_B(0)$ for any $x \in R$, then $\mu_B(x + y) = \mu_A(0) \wedge \mu_B(x + y) = (\mu_A \times \mu_B)(0, x + y) = (\mu_A \times \mu_B)((0, x) + (0, y)) \geq \{(\mu_A \times \mu_B)(0, x) \wedge (\mu_A \times \mu_B)(0, y)\} = \{\mu_A(0) \wedge \mu_B(x)\} \wedge \{\mu_A(0) \wedge \mu_B(y)\} = \{\mu_B(x) \wedge \mu_B(y)\}$. Similarly $\lambda_B(x + y) \leq \{\lambda_B(x) \vee \lambda(y)\}$.

Further for all $\alpha \in \Gamma$, $\mu_B(x\alpha y) = \{\mu_A(0) \wedge \mu_B(x\alpha y)\} = (\mu_A \times \mu_B)(0, x\alpha y) = (\mu_A \times \mu_B)((0,x)\alpha(0,y)) \geq (\mu_A \times \mu_B)(0,y) = \{\mu_A(0) \wedge \mu_B(y)\} = \mu_B(y)$ and $\lambda_B(x\alpha y) = \{\lambda_A(0) \vee \lambda_B(x\alpha y)\} = (\lambda_A \times \lambda_B)(0, x\alpha y) = (\lambda_A \times \lambda_B)((0,x)\alpha(0,y)) \leq (\lambda_A \times \lambda_B)(0,y) = \{\lambda_A(0) \vee \lambda_B(y)\} = \lambda_B(y)$, for all $x, y \in R$. Hence B is an intuitionistic fuzzy left ideals of R. Now let $a, b, x \in R$ be such that $x + a = b$. Then $(0,x) + (0,a) = (0,b)$ and so $\mu_B(x) = \{\mu_A(0) \wedge \mu_B(x)\} = (\mu_A \times \mu_B)(0,x) \geq \{(\mu_A \times \mu_B)(0,a) \wedge (\mu_A \times \mu_B)(0,b)\} = \{\{\mu_A(0) \wedge \mu_B(a)\} \wedge \{\mu_A(x) \wedge \mu_B(b)\}\} = \{\mu_B(a) \wedge \mu_B(a)\}$. Similarly, $\lambda_B(x) \leq \{\lambda_B(a) \vee \lambda_B(b)\}$. Hence B is an intuitionistic subtractive fuzzy left ideals of R.

**(ii)** Assume that $\mu_A(x) \leq \mu_A(0)$, $\lambda_A(x) \geq \lambda_A(0)$ for all $x \in R$ and $\mu_B(y) > \mu_A(0)$, $\lambda_B(y) \leq \lambda_A(0)$ for some $y \in R$. Then $\mu_B(0) \geq \mu_B(x) > \mu_A(0)$ *and* $\lambda_B(0) \leq \lambda_B(x) < \lambda_A(0)$, since $\mu_A(0) \geq \mu_A(x)$, $\lambda_A(0) \leq \lambda_A(x)$. Hence $(\mu_A \times \mu_B)(0,x) = \{\mu_A(x) \wedge \mu_B(0)\} = \mu_A(x)$. Similarly $(\lambda_A \times \lambda_B)(0,x) = \{\lambda_A(x) \vee \lambda_B(0)\} = \lambda_A(x)$ for all $x \in R$. Now it is easy to verify that $\mu_A(x+y) \geq \{\mu_A(x) \wedge \mu_A(y)\}$ and $\lambda_A(x+y) \leq \{\lambda_A(x) \vee \lambda_A(y)\}$. Again

$$\begin{aligned}
\mu_A(x\alpha y) &= (\mu_A \times \mu_B)(x\alpha y, 0)\\
&= (\mu_A \times \mu_B)((x,0)\alpha(y,0))\\
&\geq \{(\mu_A \times \mu_B)(x,0) \wedge (\mu_A \times \mu_B)(y,0)\}\\
&= \mu_B(y), \quad \alpha \in \Gamma.
\end{aligned}$$

$$\begin{aligned}
\lambda_A(x\alpha y) &= (\lambda_A \times \lambda_B)(x\alpha y, 0)\\
&= (\lambda_A \times \lambda_B)((x,0)\alpha(y,0))\\
&\leq \{(\lambda_A \times \lambda_B)(x,0) \vee (\lambda_A \times \lambda_B)(y,0)\}\\
&= \lambda_B(y) \text{ for all } x, y \in R \text{ and } \alpha \in \Gamma.
\end{aligned}$$

Now let $a, b \in A$, $x \in R$ be such that $x + a = b$. Then $(x,0) + (a,0) = (b,0)$. By similar arguments of above part (i) we find $\mu_A(x) \geq \{\mu_A(a) \wedge \mu_B(b)\}$ and $\lambda_A(x) \leq \{\lambda_A(a) \vee \lambda_B(b)\}$. Hence A is an intuitionistic subtractive fuzzy left ideals of R.

**Theorem 3.6.** Let R be a $\Gamma-$ semiring. A be IFS in R and $A_R$ be the SIFR on R. Then A is an intuitionistic subtractive fuzzy left ideal of R if and only if $A_R$ is an intuitionistic subtractive fuzzy left ideals of R.

**Proof.** Let $A = (\mu, \lambda)$ be an intuitionistic subtractive fuzzy left ideals of R. Let $(x_1, x_2), (y_1, y_2) \in R \times R$. Then clearly $\mu_{A_R}((x_1,x_2) + (y_1,y_2)) \geq \{\mu_{A_R}(x_1,x_2) \wedge \mu_{A_R}(y_1,y_2)\}$ and $\lambda_{A_R}((x_1,x_2) + (y_1,y_2)) \leq \{\lambda_{A_R}(x_1,x_2) \vee \lambda_{A_R}(y_1,y_2)\}$.

Further for all $\alpha \in \Gamma$

$$\begin{aligned}
\mu_{A_R}((x_1,x_2)\alpha(y_1,y_2)) &= \mu_{A_R}(x_1\alpha y_1, x_2\alpha y_2)\\
&= \{\mu(x_1\alpha y_1) \wedge \mu(x_2\alpha y_2)\}\\
&\geq \{\mu(y_1) \wedge \mu(y_2)\}\\
&= \mu_{A_R}(y_1, y_2).
\end{aligned}$$

Similarly

$$\lambda_{A_R}\big((x_1,x_2)\alpha(y_1,y_2)\big) = \lambda_{A_R}(x_1\alpha y_1, x_2\alpha y_2)$$

$$= \{\lambda(x_1\alpha y_1) \vee \lambda(x_2\alpha y_2)\}$$

$$\leq \{\lambda(y_1) \vee \lambda(y_2)\}$$

$$= \lambda_{A_R}(y_1, y_2).$$

Let $(a_1, a_2), (b_1, b_2) \in R \times R$ be such that $(x_1, x_2) + (a_1, a_2) = (b_1, b_2)$. Then $(x_1 + a_1,\ x_2 + a_2) = (b_1, b_2)$, it follows that $x_1 + a_1 = b_1$ and $x_2 + a_2 = b_2$. Thus

$$\mu_{A_R}(x_1, x_2) = \{\mu(x_1) \wedge \mu(x_2)\}$$

$$\geq \{\{\mu(a_1) \wedge \mu(b_1)\} \wedge \{\mu(a_2) \wedge \mu(b_2)\}\}$$

$$= \{\{\mu(a_1) \wedge \mu(a_2)\} \wedge \{\mu(b_1) \wedge \mu(b_2)\}$$

$$= \{\mu_{A_R}(a_1, a_2) \wedge \mu_{A_R}(b_1, b_2)\}.$$

Similarly, $\lambda_{A_R}(x_1, x_2) = \{\lambda(x_1) \vee \lambda(x_2)\}$

$$\leq \{\{\lambda(a_1) \vee \lambda(b_1)\} \vee \{\lambda(a_2) \vee \lambda(b_2)\}$$

$$= \{\{\lambda(a_1) \vee \lambda(a_2)\} \vee \{\lambda(b_1) \vee \lambda(b_2)\}$$

$$= \{\lambda_{A_R}(a_1, a_2) \vee \lambda_{A_R}(b_1, b_2)\}.$$

Hence $A_R$ is an intuitionistic subtractive fuzzy left ideals of R.

Conversely, let that $A_R$ is an intuitionistic subtractive fuzzy left ideals of $R \times R$. Let $x_1, x_2, y_1, y_2 \in R$.Then

$$\{\mu(x_1 + y_1) \wedge \mu(x_2 + y_2)\} = \mu_{A_R}(x_1 + y_1, x_2 + y_2)$$

$$\geq \{\mu_{A_R}(x_1, x_2) \wedge \mu_{A_R}(y_1, y_2)\}$$

$$= \{\{\mu(x_1) \wedge \mu(x_2)\} \wedge \{\mu(y_1) \wedge \mu(y_2)\}\}$$

Thus $\mu(x_1 + y_1) \geq \{\{\mu(x_1) \wedge \mu(x_2)\} \wedge \{\mu(y_1) \wedge \mu(y_2)\}\}$.

Similarly,

$$\{\lambda(x_1 + y_1) \vee \lambda(x_2 + y_2)\} = \lambda_{A_R}(x_1 + y_1, x_2 + y_2)$$

$$\leq \{\{\lambda(x_1) \vee \lambda(y_1)\} \vee \{\lambda(x_2) \vee \lambda(y_2)\}\}$$

$$= \{\{\lambda(x_1) \vee \lambda(x_2)\} \vee \{\lambda(y_1) \vee \lambda(y_2)\}\}$$

So $\lambda(x_1 + y_1) \leq \{\{\lambda(x_1) \vee \lambda(x_2)\} \vee \{\lambda(y_1) \vee \lambda(y_2)\}\}$. Now let us choose the values of $x_1, x_2, y_1, y_2$ such that $x_1 = x, x_2 = 0$ , $y_1 = y$ and $y_2 = 0$. Then

$\mu(x+y) \geq \{\{\mu(x) \wedge \mu(0)\} \wedge \{\mu(y) \wedge \mu(0)\}\} = \{\mu(x) \wedge \mu(y)\}$ and

$\lambda(x+y) \leq \{\{\lambda(x) \vee \lambda(0)\} \vee \{\lambda(y) \vee \lambda(0)\}\} = \{\lambda(x) \vee \lambda(y)\}$ (c.f. theorem 3.2.)

Again for all $\alpha \in \Gamma$, we have

$\{\mu(x_1 \alpha y_1) \wedge \mu(x_2 \alpha y_2)\} = \mu_{A_R}(x_1 \alpha y_1, x_2 \alpha y_2) \geq \mu_{A_R}(y_1, y_2) = \{\mu(y_1) \wedge \mu(y_2)\}$.

Similarly

$\{\lambda(x_1 \alpha y_1) \vee \lambda(x_2 \alpha y_2) = \lambda_{A_R}(x_1 \alpha y_1, x_2 \alpha y_2) = \lambda_{A_R}\big((x_1, x_2)\alpha(y_1, y_2)\big) \leq \lambda_{A_R}(y_1, y_2)$

$= \{\lambda(y_1) \vee \lambda(y_2)\}$ and so $\mu(x_1 \alpha y_1) \geq \{\mu(y_1) \wedge \mu(y_2)\}$.

Again by taking $x_1 = x$, $y_1 = y$ and $y_2 = 0$ and using Theorem 3.2, we get $\mu(x\alpha y) \geq \{\mu(y) \wedge \mu(0)\} = \mu(y)$ and $\lambda(x\alpha y) \leq \{\lambda(y) \vee \lambda(0)\} = \lambda(y)$. Hence A is an intuitionistic fuzzy left ideal of R. Now let $a, b, x \in R$ be such that $x + a = b$. Then $(x, 0) + (a, 0) = (b, 0)$. Since $A_R$ is an intuitionistic subtractive fuzzy left ideals of $R \times R$. It follows from theorem 3.2 that

$\mu(x) = \{\mu(x) \wedge \mu(0)\} = \mu_{A_R}(x, 0) \geq \{\mu_{A_R}(a, 0) \wedge \mu_{A_R}(b, 0)\} = \{\{\mu(a) \wedge \mu(0)\} \wedge \{\mu(b) \wedge \mu(0)\}\} = \{\mu(a) \wedge \mu(b)\}$.

Similarly

$\lambda(x) = \{\lambda(x) \vee \lambda(0)\} = \lambda_{A_R}(x, 0) \leq \{\lambda_{A_R}(a, 0) \vee \lambda_{A_R}(b, 0)\} = \{\{\lambda(a) \vee \lambda(0)\} \vee \{\lambda(b) \vee \lambda(0)\}\} = \{\lambda(a), \lambda(b)\}$. Hence, A is an intuitionistic subtractive fuzzy left ideals of R.

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