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# An importance of Linear Algebra & Matrix in Mathematics

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**Abstract:** Linear algebra allows us to start understanding basic linear systems with use of matrices and vectors. Linear algebra is the branch of mathematics concerned with the study of vectors, vector spaces (also called linear spaces), linear maps (also called linear transformations), and systems of linear equations. Vector spaces are a central theme in modern mathematics; thus, linear algebra is widely used in both abstract algebra and functional analysis. Linear algebra also has a concrete representation in analytic geometry and it is generalized in operator theory. It has extensive applications in the natural sciences and the social sciences, since nonlinear models can often be approximated by linear ones.

**Keywords:** Linear Algebra, Matrix, Linear Spaces, n- Tuples, Vectors, Linear Equation.

## I. INTRODUCTION

Linear algebra had its beginnings in the study of vectors in Cartesian 2-space and 3-space. A vector, here, is a directed line segment, characterized by both its magnitude, represented by length, and its direction. Vectors can be used to represent physical entities such as forces, and they can be added to each other and multiplied with scalars, thus forming the first example of a real vector space. Modern linear algebra has been extended to consider spaces of arbitrary or infinite dimension. A vector space of dimension $n$ is called an $n$-space. Most of the useful results from 2- and 3-space can be extended to these higher dimensional spaces. Although people cannot easily visualize vectors in $n$-space, such vectors or $n$-tuples are useful in representing data. Since vectors, as $n$-tuples, are *ordered* lists of $n$ components, it is possible to summarize and manipulate data efficiently in this framework. For example, in economics, one can create and use, say, 8-dimensional vectors or 8-tuples to represent the Gross National Product of 8 countries. One can decide to display the GNP of 8 countries for a particular year, where the countries' order is specified, for example, (United States, United Kingdom, France, Germany, Spain, India, Japan, Australia), by using a vector ($v_1$, $v_2$, $v_3$, $v_4$, $v_5$, $v_6$, $v_7$, $v_8$) where each country's GNP is in its respective position. A vector space (or linear space), as a purely abstract concept about which theorems are proved, is part of abstract algebra, and is well integrated into this discipline. Some striking examples of this are the group of invertible linear maps or matrices, and the ring of linear maps of a vector space. Linear algebra also plays an important part in analysis, notably, in the description of higher order derivatives in vector analysis and the study of tensor products and alternating maps.

## II. ELEMENTARY INTRODUCTION

Linear algebra had its beginnings in the study of vectors in cartesian 2-space and 3-space. A vector, here, is a directed line segment, characterized by both its magnitude (also called length or norm) and its direction. The zero vector is an exception; it has zero magnitude and no direction. Vectors can be used to represent physical entities such as forces, and they can be added to each other and multiplied by scalars, thus forming the first example of a real vector space, where a distinction is made between "scalars", in this case real numbers, and "vectors".

Modern linear algebra has been extended to consider spaces of arbitrary or infinite dimension. A vector space of dimension *n* is called an *n*-space. Most of the useful results from 2- and 3-space can be extended to these higher dimensional spaces. Although people cannot easily visualize vectors in *n*-space, such vectors or *n*-tuples are useful in representing data. Since vectors, as *n*-tuples, consist of n ordered components, data can be efficiently summarized and manipulated in this framework. For example, in economics, one can create and use, say, 8-dimensional vectors or 8-tuples to represent the gross national product of 8 countries. One can decide to display the GNP of 8 countries for a particular year, where the countries' order is specified, for example, (United States, United Kingdom, Armenia, Germany, Brazil, India, Japan, Bangladesh), by using a vector ($v_1$, $v_2$, $v_3$, $v_4$, $v_5$, $v_6$, $v_7$, $v_8$) where each country's GNP is in its respective position.

## III. SOME USEFUL THEOREMS

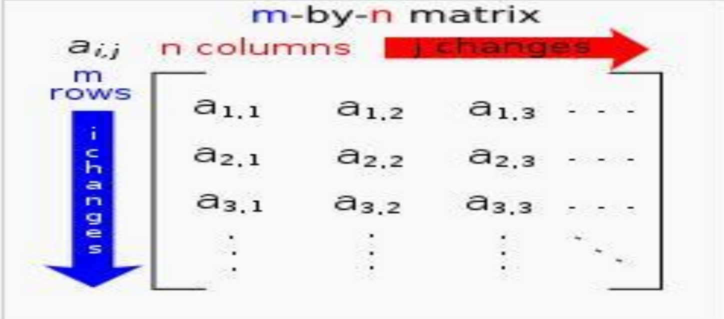
- Every vector space has a basis [1].
- Any two bases of the same vector space have the same cardinality; equivalently, the dimension of a vector space is well-defined.
- A matrix is invertible if and only if its determinant is nonzero.
- A matrix is invertible if and only if the linear map represented by the matrix is an isomorphism.
- If a square matrix has a left inverse or a right inverse then it is invertible (see invertible matrix for other equivalent statements).
- A matrix is positive semidefinite if and only if each of its eigen values is greater than or equal to zero.
- A matrix is positive definite if and only if each of its eigen values is greater than zero.
- An $n \times n$ matrix is diagonalizable (i.e. there exists an invertible matrix $P$ and a diagonal matrix $D$ such that $A = PDP^{-1}$) if and only if it has $n$ linearly independent eigenvectors.
- The spectral theorem states that a matrix is orthogonally diagonalizable if and only if it is symmetric.

For more information regarding the invertability of a matrix, consult the invertable matrix article.

## IV. LINEAR EQUATION

A linear equation is an algebraic equation in which each term is either a constant or the product of a constant and (the first power of) a single variable. Linear equations can have one or more variables. Linear equations occur abundantly in most subareas of mathematics and especially in applied mathematics. While they arise quite naturally when modeling many phenomena, they are particularly useful since many non-linear equations may be reduced to linear equations by assuming that quantities of interest vary to only a small extent from some "background" state. Linear equations do not include exponents. This article considers the case of a single equation for which one searches the real solutions. All its content applies for complex solutions and, more generally for linear equations with coefficients and solutions in any field.

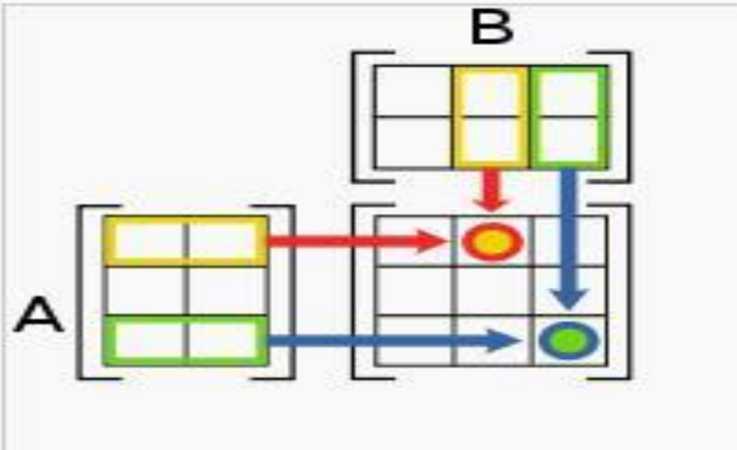
## V. MATRIX




Organization of a matrix

In mathematics, a matrix (plural matrices, or less commonly matrixes) is a rectangular array of numbers, as shown at the right. Matrices consisting of only one column or row are called vectors, while higher-dimensional, e.g. three-dimensional, arrays of numbers are called tensors. Matrices can be added and subtracted entrywise, and multiplied according to a rule corresponding to composition of linear transformations. These operations satisfy the usual identities, except that matrix multiplication is not commutative: the identity AB=BA can fail. One use of matrices is to represent linear transformations, which are higher-dimensional analogs of linear functions of the form $f(x) = cx$, where $c$ is a constant. Matrices can also keep track of the coefficients in a system of linear equations. For a square matrix, the determinant and inverse matrix (when it exists) govern the behavior of solutions to the corresponding system of linear equations, and eigen values and eigenvectors provide insight into the geometry of the associated linear transformation. Matrices find many applications. Physics makes use of them in various domains, for example in geometrical optics and matrix mechanics. The latter also led to studying in more detail matrices with an infinite number of rows and columns. Matrices encoding distances of knot points in a graph, such as cities connected by roads, are used in graph theory, and computer graphics use matrices to encode projections of three-dimensional space onto a two-dimensional screen. Matrix calculus generalizes classical analytical notions such as derivatives of functions or exponentials to matrices. The latter is a recurring need in solving ordinary differential equations. Serialism and dodecaphonism are musical movements of the 20th century that utilize a square mathematical matrix to determine the pattern of music intervals. Due to their widespread use, considerable effort has been made to develop efficient methods of matrix computing, particularly if the matrices are big. To this end, there are several matrix decomposition methods, which express matrices as products of other matrices with particular properties simplifying computations, both theoretically and practically. Sparse matrices, matrices consisting mostly of zeros, which occur, for example, in simulating mechanical experiments using the finite element method, often allow for more specifically tailored algorithms performing these tasks. The close relationship of matrices with linear transformations makes the former a key notion of linear algebra. Other types of entries, such as elements in more general mathematical fields or even rings are also used.

## VI. MATRIX MULTIPLICATION, LINEAR EQUATIONS AND LINEAR TRANSFORMATIONS




Schematic depiction of the matrix product **AB** of two matrices **A** and **B**.

Multiplication of two matrices is defined only if the number of columns of the left matrix is the same as the number of rows of the right matrix. If **A** is an m-by-$n$ matrix and **B** is an n-by-p matrix, then their matrix product **AB** is the m-by-p matrix whose entries are given by:

$$[\mathrm{AB}]_{i,j} = A_{i,1}B_{1,j} + A_{i,2}B_{2,j} + \dots + A_{i,n}B_{n,j} = \sum_{r=1}^{n} A_{i,r}B_{r,j},$$

where $1 \le i \le m$ and $1 \le j \le p$.[5] For example (the underlined entry 1 in the product is calculated as the product $1 \cdot 1 + 0 \cdot 1 + 2 \cdot 0 = 1$):

$$\begin{bmatrix} \underline{1} & \underline{0} & \underline{2} \\ -1 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & \underline{1} \\ 2 & \underline{1} \\ 1 & \underline{0} \end{bmatrix} = \begin{bmatrix} 5 & \underline{1} \\ 4 & 2 \end{bmatrix}.$$

Matrix multiplication satisfies the rules (**AB**)**C** = **A**(**BC**) (associativity), and (**A**+**B**)**C** = **AC**+**BC** as well as **C**(**A**+**B**) = **CA**+**CB** (left and right distributivity), whenever the size of the matrices is such that the various products are defined.[6] The product **AB** may be defined without **BA** being defined, namely if **A** and **B** are $m$-by-$n$ and $n$-by-$k$ matrices, respectively, and $m \neq k$. Even if both products are defined, they need not be equal, i.e. generally one has

**AB** ≠ **BA**,

*i.e.*, matrix multiplication is not commutative, in marked contrast to (rational, real, or complex) numbers whose product is independent of the order of the factors.

### *Linear Equations*

A particular case of matrix multiplication is tightly linked to *linear equations*: if **x** designates a column vector (i.e. $n$×1-matrix) of $n$ variables $x_1, x_2, \dots, x_n$, and **A** is an $m$-by-$n$ matrix, then the matrix equation

**Ax** = **b**,

where **b** is some $m$×1-column vector, is equivalent to the system of linear equations

$A_{1,1}x_1 + A_{1,2}x_2 + \dots + A_{1,n}x_n = b_1$

$A_{m,1}x_1 + A_{m,2}x_2 + \dots + A_{m,n}x_n = b_m$ .[8]

This way, matrices can be used to compactly write and deal with multiple linear equations, i.e. systems of linear equations.

### *Linear Transformation*

Matrices and matrix multiplication reveal their essential features when related to linear transformations, also known as linear maps. A real m-by-n matrix **A** gives rise to a linear transformation $\mathbf{R}^n \rightarrow \mathbf{R}^m$ mapping each vector **x** in $\mathbf{R}^n$ to the (matrix) product **Ax**, which is a vector in $\mathbf{R}^m$. Conversely, each linear transformation f: $\mathbf{R}^n \rightarrow \mathbf{R}^m$ arises from a unique m-by-n matrix ***A****:* explicitly, the (i, j)-entry of **A** is the $i^{th}$ coordinate of f($\mathbf{e}_j$), where $\mathbf{e}_j$ = (0,...,0,1,0,...,0) is the unit vector with 1 in the $j^{th}$ position and 0 elsewhere. The matrix **A** is said to represent the linear map f, and **A** is called the transformation matrix of f. The following table shows a number of 2-by-2 matrices with the associated linear maps of $\mathbf{R}^2$. The blue original is mapped to the green grid and shapes, the origin (0,0) is marked with a black point.

## VII. CONCLUSIONS

Linear transformations and the associated symmetries play a key role in modern physics. Chemistry makes use of matrices in various ways, particularly since the use of quantum theory to discuss molecular bonding and spectroscopy. In this we are presenting a study on the linear algebra and matrix in mathematics. A linear equation is an algebraic equation in which each term is either a constant or the product of a constant and (the first power of) a single variable. Linear equations can have one or more variables. Linear algebra is the branch of mathematics concerned with the study of vectors, vector spaces (also called linear spaces), linear maps (also called linear transformations), and systems of linear equations.

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