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ACYCLIC PATH DECOMPOSITION NUMBER OF SOME GRAPHS

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Abstract*:* A *decomposition* of a graph *G* is a collection ψ of edge-disjoint subgraphs $H_1, H_2, \ldots, H_r$ of *G* such that every edge of *G* belongs to exactly one H_i . If each H_i is a path or a cycle in *G*, then ψ is called a *path decomposition* of *G*. If each H_i is a path in *G*, then ψ is called an acyclic path decomposition of *G*. The minimum cardinality of a path decomposition (acyclic path decomposition) of *G* is called the *path decomposition number (acyclic path decomposition number)* of *G* and is denoted by $\pi_a(G)$. In this paper we investigate some proper acyclic path decomposition number and induced path decomposition number of special types of graphs.

***Index Terms* - Friendship graph, Pan graph, Tadpole graph, Pineapple graph, Path decomposition, Induced path decomposition.**

I. INTRODUCTION

Graphs can be used to model many types of relations and processes in physical, biological, social and information systems. Many practical problems can be represented by graphs. Emphasizing their application to real-world systems, the term *network* is sometimes defined to mean a graph in which attributes are associated with the vertices and edges.

There are more research problem going indepth in graph theory with lot of applications. One such concept is Decomposition of graphs. A *decomposition* of a graph is a collection of edge-disjoint subgraphs of such that every edge of belongs to exactly one. It has various kinds namely cyclic decomposition, ear decomposition, Hamiltonian decomposition, modular decomposition etc.

In 2009, S. Arumugam, I. Sahul Hamid and V. M. Abraham when he proved complete graph can be decomposed into $\left\lceil \frac{n}{2} \right\rceil$ paths and also proved star graph can be decomposed into $\left\lceil \frac{m}{2} \right\rceil$ paths. The focus of this paper is to investigate the path decomposition number and induced path decomposition number of some graphs. One can refer to [4 ,5] for the following definitions.

II. PRELIMINARIES

Definition 2.1. A **path** is a trail in which all vertices (and therefore also all edges) are distinct.

Definition 2.2. A **cycle** is a closed trail in which the origin and internal vertices are distinct.

Definition 2.3. A **decomposition** of a graph *G* is a collection ψ of edge-disjoint subgraphs $H_1, H_2, \ldots, H_r$ of *G* such that every edge of *G* belongs to exactly one H_i . If each H_i is a path or a cycle in *G*, then ψ is called a **path decomposition** of *G*. If each H_i is a path in *G*, then ψ is called an acyclic path decomposition of *G*. The minimum cardinality of a path decomposition (acyclic path decomposition) of *G* is called the **path decomposition number (acyclic path decomposition number)** of *G* and is denoted by $\pi_a(G)$.

Definition 2.4. A **friendship graph** is a graph consists of *n* triangles with exactly one common vertex is called the center.

Definition 2.5. The **pan graph** is the graph obtained by joining a cycle graph to a singleton graph with a bridge.

Definition 2.6. The **(*m*,*n*) tadepole graph** is a special type of graph consisting of a cycle graph on *m* (at least 3) vertices and a path graph on *n* vertices connected with a bridge.

Definition 2.7. The **pineapple graph** K_p^q is a graph obtained by appending *q* pendant edges to a vertex of a complete graph K_p ($p \geq 3, q \geq 1$). The pineapple graph is determined by its adjacency spectrum.

Definition 2.8. Let G = (V,E) be any graph, and let S ⊂ V be any subset of vertices of G. Then the **induced subgraph** G[S] is the graph whose vertex is S and whose edge set consists of all of the edges in E that have both endpoints in S.

Definition 2.9. An **Induced Path** in an undirected graph G is a path that is an induced subgraph of G. That is, it is a sequence of vertices in G such that each two adjacent vertices in the sequence are connected by an edge in G, and each two nonadjacent vertices in the sequence are not connected by an edge in G.

Definition 2.10. A decomposition of a graph G is a collection ψ of subgraphs $H_1, H_2, \ldots, H_r$ of G such that every edge of G belongs to exactly one H_i . If each $H_i$is an inducedpath or an induced cycle in G, then ψ is called an **inducedpath decomposition** of G. If each $H_i$is a path in G, then ψ is called an acyclic path decomposition of G. The minimum cardinality of an induced path decomposition of G is called an **inducedpath decomposition number** of G and is denoted by $\pi_{ia}(G)$.

III. Path Decomposition Number of Some Graphs

Theorem 3.1. Let $G = F_n$ be a friendship graph with $2n + 1$ vertices. Then $\pi_a(F_n) = n$, where $n \geq 2$.

Proof:

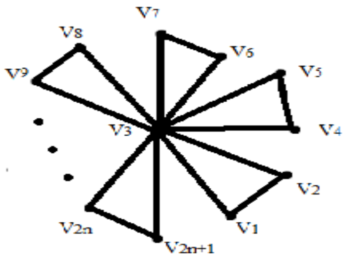


Figure 1. Friendship graph

Let the vertex set be $v_1, v_2, v_3, \ldots, v_{2n+1}$.
There are *n* cycles in which one vertex is common to all let it be v_3.
Consider the largest path in F_n is $P_1 : v_1, v_2, v_3, v_4, v_5,$
This is a path of length 4.
Next we consider a path $P_2 : v_5\ v_3\ v_6\ v_7$ of length 3.
This is a next largest path.
Similarly, we take another path $P_3 : v_7\ v_3\ v_8\ v_9$ of length 3 and so on.
$P_{n-1} : v_{2n-1}\ v_3\ v_{2n}\ v_{2n+1}$ of length 3.
At Last we have 2 edges remaining, so we take it as the last path in F_n.
(i.e,) $P_n : v_1\ v_3\ v_{2n+1}$ of length 2.
Hence F_n decomposed into n paths.
This *n* is the minimum cardinality.
Therefore path dcomposition number of F_n is *n*.Thus $\pi_a(F_n) = n$

Theorem 3.2. For a pan graph with $n + 1$ vertices. Then $\pi_a(P_n) = 2$.

Proof:

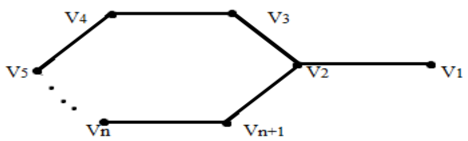


Figure 2-Pan graph

Let the vertex set be $v_1, v_2, v_3, \ldots, v_n, v_{n+1}$.

Then P_n has *n* cycle and one edge.

Path decomposition of P_n is $v_1\ v_2 v_3 \ldots\ v_n v_{n+1}$ and $v_{n+1} v_2$.

Therefore path decomposition number of P_n is 2.

Thus $\pi_a(P_n) = 2$.

Theorem 3.3. For a tadpole graph with $m + n$ vertices, then $\pi_a(T_{m,n}) = 2$.
Proof:

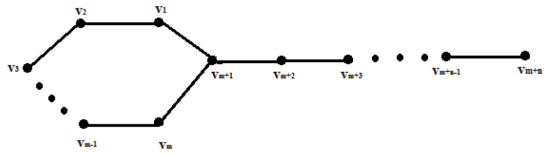


Figure 3. Tadpole graph

Let the vertex set be $v_1, v_2, v_3, \dots, v_m, v_{m+1}, \dots, v_{m+n}$.
There are m cycle and n edges.
Path decomposition of $(T_{m,n})$ is $v_{m+n}v_{m+n-1}\dots v_{m+1}v_1v_mv_{m-1}\dots v_2v_1$ and v_1v_{m+1}.
Therefore path decomposition number of $(T_{m,n})$ is 2. Thus $\pi_a(T_{m,n}) = 2$.

Theorem 3.4. For a pineapple graph K_n^m, then $\pi_a(K_n^m) = \left\lceil \frac{n}{2} \right\rceil + \left\lfloor \frac{m}{2} \right\rfloor$.

Proof:

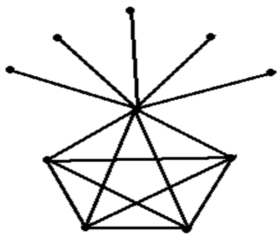


Figure 4 .A Pineapple graph

Pineapple graph is a combination of complete graph and star graph.

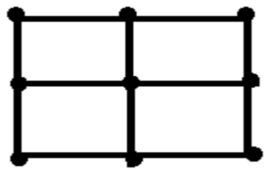
Path decomposition number of complete graph is $\left\lceil \frac{n}{2} \right\rceil$.

Path decomposition number of star graph is $\left\lfloor \frac{m}{2} \right\rfloor$.

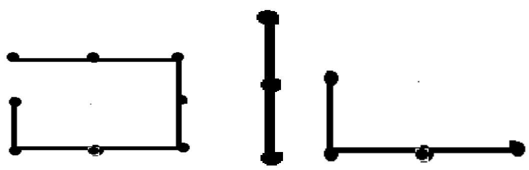
Therefore path decomposition number of pinapple graph is $\left\lceil \frac{n}{2} \right\rceil + \left\lfloor \frac{m}{2} \right\rfloor$.

Thus $\pi_a(K_n^m) = \left\lceil \frac{n}{2} \right\rceil + \left\lfloor \frac{m}{2} \right\rfloor$.

Theorem 3.5. Let G = $p_m \times p_n$ be a graph with mn vertices, then $\pi_a(p_m \times p_n) = n + m - 3$ where $m, n \geq 3$.
Proof:
We shall prove the theorem by induction on m.
Let m = 3, n = 3 then G $\cong p_3 \times p_3$.



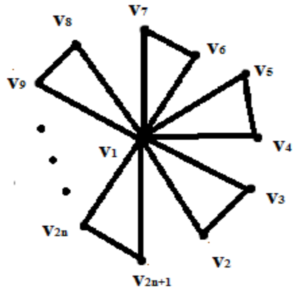
Path decompositions are



Then $\pi_a(G) = 3 = 3 + 3 - 3$
The result is true for m = 3, n = 3.
Assume that the result is true for m = k.
i.e., $\pi_a(p_k \times p_n) = n + k - 3$
Let us prove that the result for m = k + 1.
By induction hypothesis, $\pi_a(p_{k+1} \times p_n) = \pi_a(p_k \times p_n) + 1$.
$= n + k - 3 + 1$.
$= n + k - 2$
Therefore the result is true for m = k + 1.Therefore by the principle of mathematical induction $\pi_a(p_m \times p_n) = n + m - 3$ where $m, n \geq 3$.

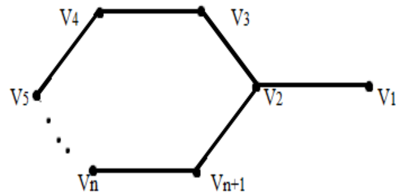
IV. INDUCED PATH DECOMPOSITION NUMBER OF SOME GRAPHS

Theorem 4.1. Let $G = F_n$ be a friendship graph with $2n + 1$ vertices. Then $\pi_{ia}(F_n) = 2n$, where $n \geq 2$.
Proof :



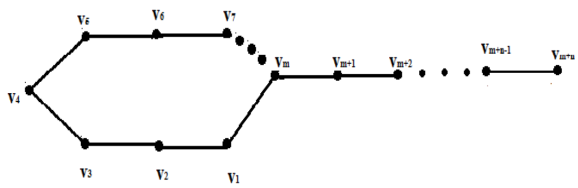
Let v_1 be the center vertex.
F_n has cycles namely $C_1, C_2, \ldots C_n$ and C_1 is a cycle of length 3.
$C_1 : v_1v_2v_3v_1$
$C_2 : v_1v_4v_5v_1$
$C_3 : v_1v_6v_7v_1$
.
.
.
$C_n : v_1v_{2n}v_{2n+1}v_1$
In friendship graph the longest path is P_3.
We take a induced path decompositions namely,
$P_1 : v_3v_1v_4$
$P_2 : v_5v_1v_6$
$P_3 : v_7v_1v_8$
.
.
.
$P_n : v_{2n+1}v_1v_2$
There are n remaining single edges.
These are totally 2n induced path decompositions.
$\pi_{ia}(G) = 2n$.

Theorem 4.2. For a pan graph with $n + 1$ vertices, then $\pi_{ia}(P_n) = 2$, where $n \geq 4$.
Proof :



Pan graph is a combination of path and cycles.
Let the vertex set be $v_1, v_2, v_3, \ldots v_n, v_{n+1}$.
Then induced path decomposition of P_n is $v_1v_2v_3\ldots v_n$ and $v_nv_{n+1}v_2$.
Therefore induced path decomposition of P_n is 2
Therefore $\pi_{ia}(P_n)$ is 2.

Theorem 4.3.
For a tadpole graph with $m + n$ vertices, then $\pi_{ia}(T_{m,n}) = 2$ where $m \geq 4$.
Proof :



Tadpole graph is a combination of path and cycles.
Let the vertex set be $v_1, v_2, v_3, \ldots v_m, v_{m+1} \ldots v_{m+m-1}, v_{m+n}$.
Then induced path decomposition of $T_{m,n}$ is $V_{m+n}v_{m+n-1}\ldots v_m\, v_{m-1}\ldots v_3v_2$ and $v_2v_1v_m$.
Therefore induced path decomposition of $T_{m,n}$ is 2.
Therefore $\pi_{ia}(T_{m,n})$ is 2,

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