CONSTRUCTING DOUBLY EVEN MAGIC SQUARES USING REVERSION PROCESS: A SIMPLE TECHNIQUE

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Abstract: In this paper, we introduce a technique for finding doubly even magic squares of order *n* (*n* divisible by 4) so termed as reversion process. An alternative method is also developed by modifying slightly in between the steps, followed by some illustrations.

Keywords **– Square matrix, magic square, reversion process, magic sum, Tomba's constant.**

I. INTRODUCTION

In a magic square of order n, the sum of the elements of each row, column, and main diagonals are equal and is denoted as magic constant. The magic constant or magic sum ***S*** exist for all order $n \geq 1$ except for $n = 2$ and is determined by the formula $S = \frac{n(n^2+1)}{2}$.

Among the various methods for constructing magic squares a simple technique developed by Tomba [5] using Basic Latin squares is considered. This method needs three steps for construction of odd order magic square and five steps [6–8] for constructing doubly even and singly even magic squares. Depending upon the choice of central block and assignment of pair number satisfying Tomba's constant T, different weak magic squares are generated that can produce different cipher text as far as possible from plaintext. Romen et al [4] developed a MATLAB algorithm for modifying slightly the technique developed by Tomba [5] for construction of odd magic squares using Basic Latin squares. In this paper we introduced a simple technique for constructing doubly-even magic squares from an initial square matrix using reversion process in three steps only.

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II. METHODOLOGY

The technique for constructing doubly even magic squares using reversion process can be expressed as follows:
Let the initial square matrix of doubly even *n* be written in the form

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}_{n\times n}.$$

Then carry out the transformation of the above square matrix by reversing alternative two consecutive pairs of columns and rows starting from a column/row at any even position $p \leq n$ for column/row and then again $p \leq n$ for row/column respectively to get the doubly even magic square of order *n*. Alternatively, carrying out the transformation by reversing two consecutive pairs of columns and rows starting from a column/row at any even position $p \leq n$ for column/row and then again $p+2$ for rows/column respectively to get the final doubly even magic square of order *n*. The final magic square satisfies the Tomba's constant $T = n^2 + 1$.

III. REVERSION PROCESS ALGORITHM

Step 1**:** Take the initial square matrix M of doubly even order *n*.
Step 2: Initialise the starting even column position $p \leq n$

Step 3: Initialise the two column pair positions c_1 and c_2 for transformation as $c_1 = p$, $c_2 = c_1 + 1$ $(if\ c_2 > n, c_2 = 1)$

Step 4: Transform M by reversing the elements of column c_1 and c_2 within the matrix itself.

Step 5: Compute the next column pair as $c_1 = c_1 + 4(if\ c_1 > n, c_1 = \mathrm{mod}(c_1, n))$, $c_2 = c_1 + 1(if\ c_2 > n, c_2 = 1)$

Step 6: Repeat step 4 and 5 for column wise transformation $\frac{n}{4}$ times.

Step 7**:** Initialise the two rows pair positions r_1 and r_2 for row wise transformation as $r_1 = p$, $r_2 = r_1 + 1$ $(if\ r_2 > n, r_2 = 1)$

Step 8: Transform M by reversing the elements of rows r_1 and r_2 within the matrix itself.

Step 9: Compute the next row pair as $r_1 = r_1 + 4(if\ r_1 > n, r_1 = \mathrm{mod}(r_1, n))$, $r_2 = r_1 + 1(if\ r_2 > n, r_2 = 1)$

Step 10: Repeat step 8 and 9 for row wise transformation $\frac{n}{4}$ times and the result will be final magic square matrix.

Step 11: Stop

Alternative Method:

The above algorithm can be modified slightly to give the desire magic square by changing the row transformation i.e from step 7 to step 10 as:

Step 7**:** Initialise the two rows pair positions r_1 and r_2 for row wise transformation as

$$r_1 = p + 2(if\ r_1 > n, r_1 = \mathrm{mod}(r_1, n)),\ r_2 = r_1 + 1\ (if\ r_2 > n, r_2 = 1)$$

Step 8: Transform M by reversing the elements of rows r_1 and r_2 within the matrix itself.

Step 9: Compute the next row pair as $r_1 = r_1 + 4(if\ r_1 > n, r_1 = \mathrm{mod}(r_1, n))$, $r_2 = r_1 + 1(if\ r_2 > n, r_2 = 1)$

Step 10: Repeat step 8 and 9 for row wise transformation $\frac{n}{4}$ times and the result will be final magic square matrix.

Step 11: Stop.

IV. ILLUSTRATIONS

Illustration 1: (4 x 4) Magic Square

Step 1: Input square matrix M (4 x 4)

	1	2	3	4	**10**
	5	6	7	8	**26**
	9	10	11	12	**42**
	13	14	15	16	**58**
34	**28**	**32**	**36**	**40**	**34**

Step2: Column transformations of matrix M

	1	14	15	4	**34**
	5	10	11	8	**34**
	9	6	7	12	**34**
	13	2	3	16	**34**
34	**28**	**32**	**36**	**40**	**34**

Step3: Row transformations of step 2 matrix [require magic square]

	1	14	15	4	**34**
	8	11	10	5	**34**
	12	7	6	9	**34**
	13	2	3	16	**34**
34	**34**	**34**	**34**	**34**	**34**

Alternative Method: (Step 1 & 2 are same)

Step 3(modified): Row transformations of step 2 matrix [require magic square]

	4	15	14	1	**34**
	5	10	11	8	**34**
	9	6	7	12	**34**
	16	3	2	13	**34**
34	**34**	**34**	**34**	**34**	**34**

Illustration 2: (8x8) Magic Square

Step1: Input square matrix M (8 x 8)

	1	2	3	4	5	6	7	8	**36**
	9	10	11	12	13	14	15	16	**100**
	17	18	19	20	21	22	23	24	**164**
	25	26	27	28	29	30	31	32	**228**
	33	34	35	36	37	38	39	40	**292**
	41	42	43	44	45	46	47	48	**356**
	49	50	51	52	53	54	55	56	**420**
	57	58	59	60	61	62	63	64	**484**
260	**232**	**240**	**248**	**256**	**264**	**272**	**280**	**288**	**260**

Step2: Column transformations of matrix M

	1	58	59	4	5	62	63	8	**260**
	9	50	51	12	13	54	55	16	**260**
	17	42	43	20	21	46	47	24	**260**
	25	34	35	28	29	38	39	32	**260**
	33	26	27	36	37	30	31	40	**260**
	41	18	19	44	45	22	23	48	**260**
	49	10	11	52	53	14	15	56	**260**
	57	2	3	60	61	6	7	64	**260**
260	**232**	**240**	**248**	**256**	**264**	**272**	**280**	**288**	**260**

Step3: Row transformations of step 2 matrix [require magic square]

	1	58	59	4	5	62	63	8	**260**
	16	55	54	13	12	51	50	9	**260**
	24	47	46	21	20	43	42	17	**260**
	25	34	35	28	29	38	39	32	**260**
	33	26	27	36	37	30	31	40	**260**
	48	23	22	45	44	19	18	41	**260**
	56	15	14	53	52	11	10	49	**260**
	57	2	3	60	61	6	7	64	**260**
260	**260**	**260**	**260**	**260**	**260**	**260**	**260**	**260**	**260**

Alternative Method: (Step 1 & 2 are same)

Step 3(modified): Row transformations of step 2 matrix [require magic square]

8	63	62	5	4	59	58	1	**260**
9	50	51	12	13	54	55	16	**260**

	17	42	43	20	21	46	47	24	**260**
	32	39	38	29	28	35	34	25	**260**
	40	31	30	37	36	27	26	33	**260**
	41	18	19	44	45	22	23	48	**260**
	49	10	11	52	53	14	15	56	**260**
	64	7	6	61	60	3	2	57	**260**
260	**260**	**260**	**260**	**260**	**260**	**260**	**260**	**260**	**260**

Illustration 3: (12x12) Magic Square

Step1: Input square matrix M (8 x 8)

	1	2	3	4	5	6	7	8	9	10	11	12	**78**
	13	14	15	16	17	18	19	20	21	22	23	24	**222**
	25	26	27	28	29	30	31	32	33	34	35	36	**366**
	37	38	39	40	41	42	43	44	45	46	47	48	**510**
	49	50	51	52	53	54	55	56	57	58	59	60	**654**
	61	62	63	64	65	66	67	68	69	70	71	72	**798**
	73	74	75	76	77	78	79	80	81	82	83	84	**942**
	85	86	87	88	89	90	91	92	93	94	95	96	**1086**
	97	98	99	100	101	102	103	104	105	106	107	108	**1230**
	109	110	111	112	113	114	115	116	117	118	119	120	**1374**
	121	122	123	124	125	126	127	128	129	130	131	132	**1518**
	133	134	135	136	137	138	139	140	141	142	143	144	**1662**
870	**804**	**816**	**828**	**840**	**852**	**864**	**876**	**888**	**900**	**912**	**924**	**936**	**870**

Step2: Column transformations of matrix M

	1	134	135	4	5	138	139	8	9	142	143	12	**870**
	13	122	123	16	17	126	127	20	21	130	131	24	**870**
	25	110	111	28	29	114	115	32	33	118	119	36	**870**
	37	98	99	40	41	102	103	44	45	106	107	48	**870**
	49	86	87	52	53	90	91	56	57	94	95	60	**870**
	61	74	75	64	65	78	79	68	69	82	83	72	**870**
	73	62	63	76	77	66	67	80	81	70	71	84	**870**
	85	50	51	88	89	54	55	92	93	58	59	96	**870**
	97	38	39	100	101	42	43	104	105	46	47	108	**870**
	109	26	27	112	113	30	31	116	117	34	35	120	**870**
	121	14	15	124	125	18	19	128	129	22	23	132	**870**
	133	2	3	136	137	6	7	140	141	10	11	144	**870**
870	**804**	**816**	**828**	**840**	**852**	**864**	**876**	**888**	**900**	**912**	**924**	**936**	**870**

Step3: Row transformations of step 2 matrix [require magic square]

1	134	135	4	5	138	139	8	9	142	143	12	**870**
24	131	130	21	20	127	126	17	16	123	122	13	**870**
36	119	118	33	32	115	114	29	28	111	110	25	**870**
37	98	99	40	41	102	103	44	45	106	107	48	**870**

	49	86	87	52	53	90	91	56	57	94	95	60	**870**
	72	83	82	69	68	79	78	65	64	75	74	61	**870**
	84	71	70	81	80	67	66	77	76	63	62	73	**870**
	85	50	51	88	89	54	55	92	93	58	59	96	**870**
	97	38	39	100	101	42	43	104	105	46	47	108	**870**
	120	35	34	117	116	31	30	113	112	27	26	109	**870**
	132	23	22	129	128	19	18	125	124	15	14	121	**870**
	133	2	3	136	137	6	7	140	141	10	11	144	**870**
870	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**

Alternative Method: (Step 1 & 2 are same)

Step 3(modified): Row transformations of step 2 matrix [require magic square]

	12	143	142	9	8	139	138	5	4	135	134	1	**870**
	13	122	123	16	17	126	127	20	21	130	131	24	**870**
	25	110	111	28	29	114	115	32	33	118	119	36	**870**
	48	107	106	45	44	103	102	41	40	99	98	37	**870**
	60	95	94	57	56	91	90	53	52	87	86	49	**870**
	61	74	75	64	65	78	79	68	69	82	83	72	**870**
	73	62	63	76	77	66	67	80	81	70	71	84	**870**
	96	59	58	93	92	55	54	89	88	51	50	85	**870**
	108	47	46	105	104	43	42	101	100	39	38	97	**870**
	109	26	27	112	113	30	31	116	117	34	35	120	**870**
	121	14	15	124	125	18	19	128	129	22	23	132	**870**
	144	11	10	141	140	7	6	137	136	3	2	133	**870**
870	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**

V. CONCLUSION

The simple and easy method or alternative method described as reversion process algorithm can be used for constructing a doubly even magic square of any order n (divisible by 4) and $T = n^2 + 1$ (Tomba's constant) can be used for checking the accuracy of the magic square.

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