Sum Square Difference Product Prime Labeling of Some Planar Graphs

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Abstract: Sum square difference product prime labeling of a graph is the labeling of the vertices with {0,1,2---------,p-1} and the edges with absolute difference of the square of the sum of the labels of the incident vertices and product of the labels of the incident vertices. The greatest common incidence number of a vertex (***gcin***) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the ***gcin*** of each vertex of degree greater than one is one, then the graph admits sum square difference product prime labeling. Here we identify some planar graphs for sum square difference product prime labeling.

***IndexTerms* - Prime labeling, greatest common incidence number, sum square, planar graphs.**

I. Introduction

All graphs in this paper are planar. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4] . Some basic concepts are taken from [1] and [2]. In [5], we introduced the sum square difference product prime labeling and proved the result for some path related graphs.In this paper we investigated sum square difference product prime labeling of some planar graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

II. MAIN RESULT

Definition 2.1 Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection f : V(G) → {0,1,2,3,-----------,p-1} by $f(v_i)$ = i−1 , for every i from 1 to p and define a 1-1 mapping f_{ssdppl}^* : E(G) → set of natural numbers N by $f_{ssdppl}^*(uv)$ = $|\{f(u)+f(v)\}^2 - f(u)f(v)|$. The induced function f_{ssdppl}^* is said to be sum square difference product prime labeling, if for each vertex of degree at least 2, the greatest common incidence number is 1.

Definition 2.2 A graph which admits sum square difference product prime labeling is called a sum square difference product prime graph.

Theorem 2.1 Triangular belt TB(↑↑ − − −−↑) admits sum square difference product prime labeling.

Proof: Let G = TB(↑↑ − − −−↑) and let v_1,v_2,----------------,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| = 4n-3

Define a function f : V → {0,1,2,3,----------------,2n-1 } by

$f(v_i)$ = i-1 , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssdppl}^* is defined as follows

$f_{ssdppl}^*(v_i\, v_{i+1})$	= $3i^2-3i+1$,	i = 1,2,------------,2n-1
$f_{ssdppl}^*(v_{2i-1}\, v_{2i+1})$	= $12i^2-12i+4$,	i = 1,2,------------,n-1
$f_{ssdppl}^*(v_{2i}\, v_{2i+2})$	= $12i^2+1$,	i = 1,2,------------,n-1

Clearly f_{ssdppl}^* is an injection.

gcin of (v_1) = gcd of {1,4} = 1

gcin of (v_{i+1}) = gcd of $\{f_{ssdppl}^*(v_i\, v_{i+1}), f_{ssdppl}^*(v_{i+1}\, v_{i+2})\}$
= gcd of { $3i^2-3i+1$, $3i^2+3i+1$}
= gcd of {6i , $3i^2-3i+1$}
= gcd of {3i , $3i^2-3i+1$}
= gcd of {3i , 3i(i-1)+1}
= 1, i = 1,2,------------,2n-2

gcin of (v_{2n}) = gcd of $\{f_{ssdppl}^*(v_{2n-2}\, v_{2n}), f_{ssdppl}^*(v_{2n-1}\, v_{2n})\}$

= gcd of { $12n^2$-24n+13, $12n^2$-18n+7}
= gcd of { $12n^2$-24n+13, 6n-6}
= gcd of { (6n-6), (2n-2)(6n-6)+1}
= 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence TB(↑↑ $- - --$↑) , admits sum square difference product prime labeling. ■

Example 2.1 G = TB(↑↑↑↑)

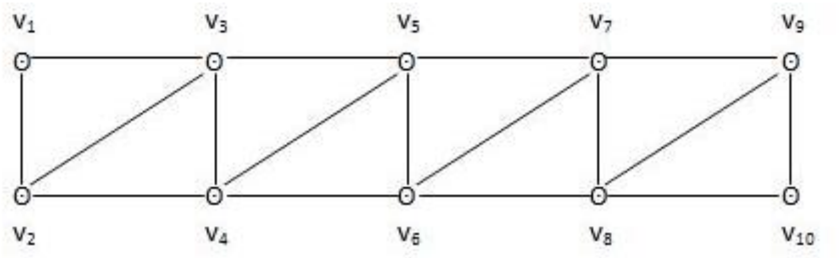

fig- 2.1

Theorem 2.2 Triangular belt TB(↑↓↑↓ $- - --$) admits sum square difference product prime labeling.

Proof: Let G = TB(↑↓↑↓ $- - --$) and let v_1,v_2,----------------,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| = 4n-3

Define a function f : V → {0,1,2,3,----------------,2n-1 } by

$f(v_i)$ = i-1 , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(v_i\ v_{i+1})$ = $3i^2$-3i+1, i = 1,2,------------,2n-1

Case(i) n is odd

$f^*_{ssdppl}(v_{4i-3}\ v_{4i})$ = $48i^2$-60i+21, i = 1,2,------------,$\frac{n-1}{2}$

$f^*_{ssdppl}(v_{4i-1}\ v_{4i+2})$ = $48i^2$-12i+3, i = 1,2,------------,$\frac{n-1}{2}$

$f^*_{ssdppl}(v_{4i-2}\ v_{4i})$ = $48i^2$-48i+13, i = 1,2,------------,$\frac{n-1}{2}$

$f^*_{ssdppl}(v_{4i}\ v_{4i+2})$ = $48i^2$+1, i = 1,2,------------,$\frac{n-1}{2}$

Case (ii) n is even

$f^*_{ssdppl}(v_{4i-3}\ v_{4i})$ = $48i^2$-60i+21, i = 1,2,------------,$\frac{n}{2}$

$f^*_{ssdppl}(v_{4i-1}\ v_{4i+2})$ = $48i^2$-12i+3, i = 1,2,------------,$\frac{n-2}{2}$

$f^*_{ssdppl}(v_{4i-2}\ v_{4i})$ = $48i^2$-48i+13, i = 1,2,------------,$\frac{n}{2}$

$f^*_{ssdppl}(v_{4i}\ v_{4i+2})$ = $48i^2$+1, i = 1,2,------------,$\frac{n-2}{2}$

Clearly f^*_{ssdppl} is an injection.

gcin of (v_1) = gcd of {1,9} = 1

gcin of (v_{i+1}) = gcd of {$f^*_{ssdppl}(v_i\ v_{i+1})$, $f^*_{ssdppl}(v_{i+1}\ v_{i+2})$ }
= 1, i = 1,2,------------,2n-2

gcin of (v_{2n}) = gcd of {$f^*_{ssdppl}(v_{2n-2}\ v_{2n})$, $f^*_{ssdppl}(v_{2n-1}\ v_{2n})$ }
= 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence TB(TB(↑↓↑↓ $- - --$) , admits sum square difference product prime labeling. ■

Example 2.2 G = TB(↑↓↑↓)

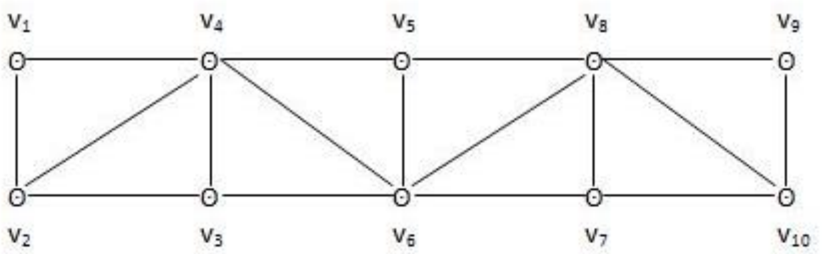

fig- 2.2

Theorem 2.3 The graph P_2+N_m admits sum square difference product prime labeling.

Proof: Let G be the graph and let v_1,v_2,----------------,v_{m+2} are the vertices of G
Here |V(G)| = m+2 and |E(G)| = 2m+1
Define a function f : V → {0,1,2,3,----------------,m+1 } by

$f(v_i) = i+1$, i = 1,2,------,m
f(a) = 0, f(b) = 1.

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(av_i)$ = $(i+1)^2$, i = 1,2,------------,m
$f^*_{ssdppl}(bv_i)$ = i^2+3i+3, i = 1,2,------------,m
$f^*_{ssdppl}(ab)$ = 1.

Clearly f^*_{ssdppl} is an injection

gcin of (a) = 1.
gcin of (b) = 1.
gcin of (v_i) = gcd of $\{f^*_{ssdppl}(av_i), f^*_{ssdppl}(bv_i)\}$
= gcd of { $(i+1)^2$, i^2+3i+3}
= gcd of {i+1 , (i+1)(i+2)+1}
= 1, i = 1,2,------------,m

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence P_2+N_m, admits sum square difference product prime labeling. ■

Example 2.3 G = P_2+N_4

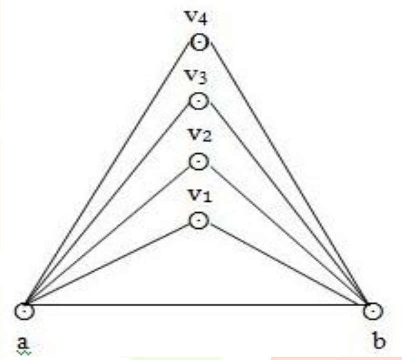

fig – 2.3

Theorem 2.4 The graph $PL_2(n)$ admits sum square difference product prime labeling.
Proof: Let G = $PL_2(n)$ and let v_1,v_2,----------------,v_{n+2} are the vertices of G
Here |V(G)| = n+2 and |E(G)| = 3n
Define a function f : V → {0,1,2,3,----------------,n+1 } by

$f(v_i) = i+1$, i = 1,2,------,n
f(a) = 0.
f(b) = 1.

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(av_i)$ = $(i+1)^2$, i = 1,2,------------,n
$f^*_{ssdppl}(bv_i)$ = i^2+3i+3, i = 1,2,------------,n
$f^*_{ssdppl}(v_i v_{i+1})$ = $3i^2+9i+7$, i = 1,2,------------,n-1
$f^*_{ssdppl}(ab)$ = 1.

Clearly f^*_{ssdppl} is an injection

gcin of (a) = 1,
gcin of (b) = 1.
gcin of (v_i) = gcd of $\{f^*_{ssdppl}(av_i), f^*_{ssdppl}(bv_i)\}$
= 1, i = 1,2,------------,n.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $PL_2(n)$, admits sum square difference product prime labeling. ■

Example 2.4 G = $PL_2(4)$

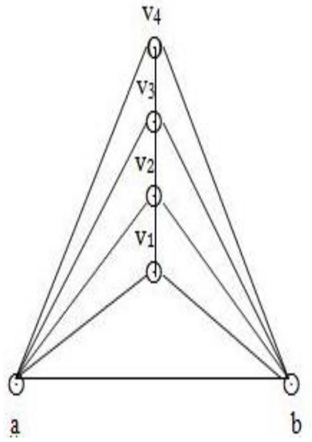

fig – 2.4

Theorem 2.5 Jewel graph J_n admits sum square difference product prime labeling.

Proof: Let G = J_n and let v_1, v_2,----------------,v_{n+4} are the vertices of G

Here |V(G)| = n+4 and |E(G)| = 2n+5

Define a function f : V → {0,1,2,3,----------------,n+3 } by

$f(v_i) = i+3$, i = 1,2,------,n

f(a) = 0, f(b) = 1, f(x) = 2, f(y) = 3

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(av_i)$ = $(i+3)^2$, i = 1,2,------------,n

$f^*_{ssdppl}(bv_i)$ = $i^2+7i+13$, i = 1,2,------------,n

$f^*_{ssdppl}(ax)$ = 4.

$f^*_{ssdppl}(ay)$ = 9.

$f^*_{ssdppl}(bx)$ = 7.

$f^*_{ssdppl}(by)$ = 13.

$f^*_{ssdppl}(xy)$ = 19.

Clearly f^*_{ssdppl} is an injection

gcin of (a) = 1.

gcin of (b) = 1.

gcin of (x) = gcd of {4,7) = 1.

gcin of (y) = gcd of {9,13) = 1.

gcin of (v_i) = gcd of {$f^*_{ssdppl}(av_i)$, $f^*_{ssdppl}(bv_i)$ }

= gcd of {$(i+3)^2$, $i^2+7i+13$}

= gcd of {(i+3), (i+3)(i+4)+1}= 1, i = 1,2,------------,n

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence J_n, admits sum square difference product prime labeling. ■

Example 2.5 G = J_3

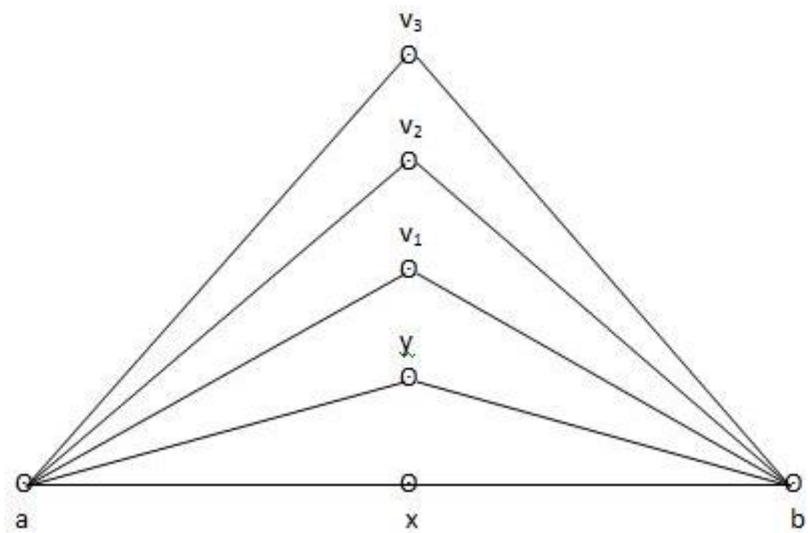

fig – 2.5

Theorem 2.6 Jelly fish graph JF(m,n) admits sum square difference product prime labeling.

Proof: Let G = JF(m,n) and let v_1, v_2,----------------,v_{n+m+4} are the vertices of G
Here |V(G)| = n+m+4 and |E(G)| = n+m+5
Define a function f : V → {0,1,2,3,----------------,m+n+3 } by

$f(v_i) = i+3$, i = 1,2,------,m

$f(u_i) = m+i+3$, i = 1,2,------,n

f(a) = 0, f(b) = 1, f(x) = 2, f(y) = 3

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(av_i)$ = $(i+3)^2$, i = 1,2,------------,m
$f^*_{ssdppl}(bu_i)$ = $(m+i+4)^2-(m+i+3)$, i = 1,2,------------,n
$f^*_{ssdppl}(ax)$ = 4.
$f^*_{ssdppl}(ay)$ = 9.
$f^*_{ssdppl}(bx)$ = 7.
$f^*_{ssdppl}(by)$ = 13.
$f^*_{ssdppl}(xy)$ = 19.
Clearly f^*_{ssdppl} is an injection
gcin of (a) = 1.
gcin of (b) = 1.
gcin of (x) = 1.
gcin of (y) = 1.
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence JF(m,n), admits sum square difference product prime labeling. ■

Example 2.6 G = JF(3,4)

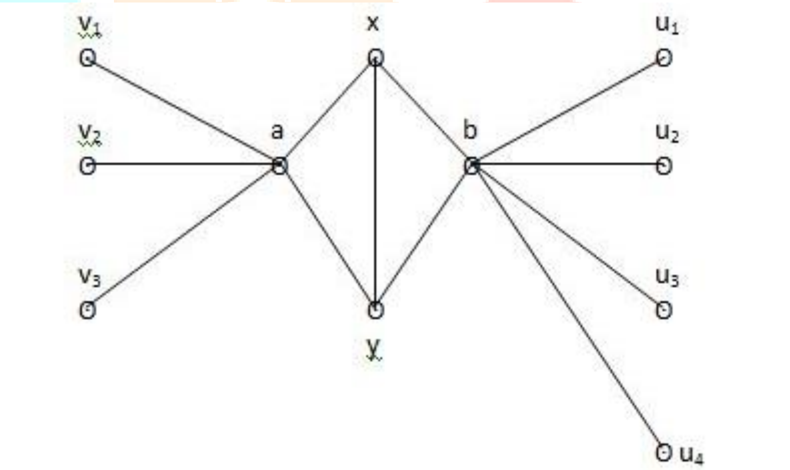

fig – 2.6

Theorem 2.7 Two copies of cycle C_n sharing a common edge admits sum square difference product prime labeling.

Proof :- Let G = $2(C_n)$ – e and let v_1, v_2,----------------,v_{2n-2} are the vertices of G.

Here |V(G)| = 2n-2 and |E(G)| = 2n-1
Define a function f : V → {0,1,2,3,----------------,2n-3} by

$f(v_i) = i-1$, i = 1,2,------,2n-2

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(v_i\, v_{i+1})$ = $3i^2-3i+1$, i = 1,2,------------,2n-3
$f^*_{ssdppl}(v_1\, v_{2n-2})$ = $(2n-3)^2$.

Case(i) n is even.

$f^*_{ssdppl}(v_{(\frac{n}{2})}\, v_{(\frac{3n-2}{2})})$ = $\frac{13n^2-38n+28}{4}$

Case(ii) n is odd.

$f^*_{ssdppl}(v_{(\frac{n+1}{2})}\, v_{(\frac{3n-1}{2})})$ = $\frac{13n^2-26n+13}{4}$

Clearly f^*_{ssdppl} is an injection
gcin of (v_1) = gcd of {$f^*_{ssdppl}(v_1\, v_2)$, $f^*_{ssdppl}(v_1\, v_{2n-2})$ }
= gcd of { 1, $(2n-3)^2$} = 1.
gcin of (v_{i+1}) = 1, i = 1,2,-------------,2n-4.
gcin of (v_{2n-2}) = gcd of {$f^*_{ssdppl}(v_1\, v_{2n-2})$, $f^*_{ssdppl}(v_{2n-3}\, v_{2n-2})$ }

= gcd of { $(2n-3)^2$, $12n^2-42n+37$}
= gcd of { (2n-3), (2n-3)(6n-12)+1}= 1.
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $2(C_n)$ – e, admits sum square difference product prime labeling. ■
Example 2.7 G = $2(C_5)$ –e

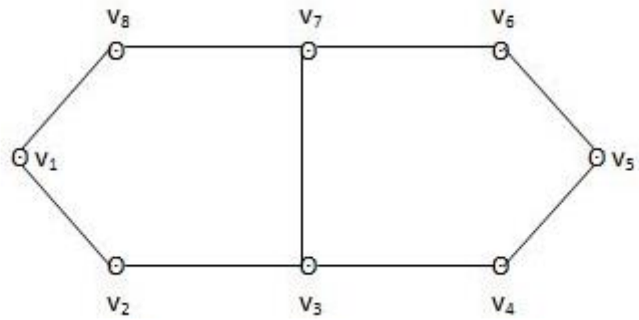

fig -2.7

Theorem 2.8 Two copies of cycle C_n sharing a common vertex admits sum square difference product prime labeling , when (n-2) ≢ 0 (mod 7) and (n-4) ≢ 0 (mod 7).

Proof : Let G = $2(C_n)$ – v and let v_1,v_2,----------------,v_{2n-1} are the vertices of G.

Here |V(G)| = 2n-1 and |E(G)| = 2n
Define a function f : V → {0,1,2,3,----------------,2n-2} by
$f(v_i)$ = i-1 , i = 1,2,------,2n-1
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(v_i\ v_{i+1})$ = $3i^2-3i+1$, i = 1,2,------------,2n-2
$f^*_{ssdppl}(v_1\ v_n)$ = $(n-1)^2$.
$f^*_{ssdppl}(v_n\ v_{2n-1})$ = $7n^2-14n+7$.
Clearly f^*_{ssdppl} is an injection
gcin of (v_1) = gcd of {$f^*_{ssdppl}(v_1\ v_2)$, $f^*_{ssdppl}(v_1\ v_n)$ }
= gcd of { 1, $(n-1)^2$} = 1.
gcin of (v_{i+1}) = 1, i = 1,2,-------------,2n-3.
gcin of (v_{2n-1}) = gcd of {$f^*_{ssdppl}(v_n\ v_{2n-1})$, $f^*_{ssdppl}(v_{2n-1}\ v_{2n-2})$ }
= gcd of { $7n^2-14n+7$, $12n^2-30n+19$}
= 1.
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $2(C_n)$ – v, admits sum square difference product prime labeling. ■
Example 2.8 G = $2(C_5)$ – v

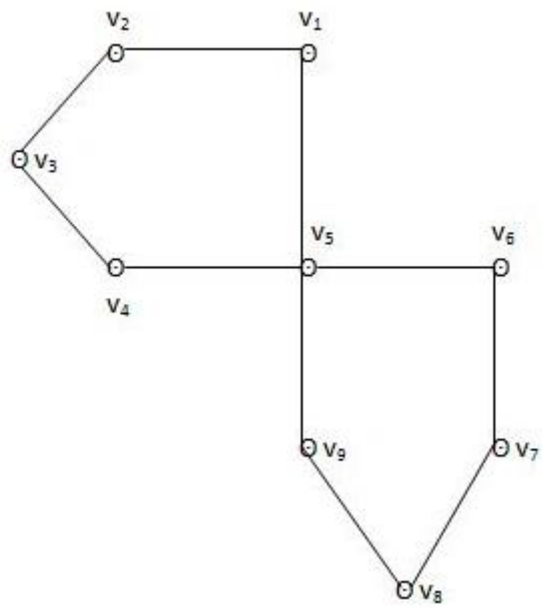

fig – 2.8

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