

# A Comprehensive Study On Buckling Analysis Of Composite Laminated Plates

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## ABSTRACT :

When it comes to establishing an appropriate accurate, analytical, or semi-analytical approach for studying buckling loads on laminated plates, it was discovered that the impacts of boundary conditions and lamination arrangements (i.e. stacking sequence and orientation of a lamina) are significant aspects to take into consideration. Additionally, it was discovered that the precision of stresses, strains, buckling loads, and other similar quantities becomes more accurate as the derivative order of shear deformation grows, and this occurs without the necessity for shear correction factors.

**Keywords:** Literature review, Buckling, Laminated Composite, Buckling Loads.

## 1. INTRODUCTION

There is a wide range of current engineering application domains that make extensive use of composite materials. These fields include conventional disciplines such as autos, robots, everyday appliances, the construction industry, and so on, as well as new space industries. This is as a result of their very high strength to weight ratio, as well as their modulus to weight ratio, and the fact that the structural qualities can be controlled by varying the fiber orientation, stacking scheme, and the number of laminates.

A significant amount of attention has been paid to the mechanical behavior of rectangular laminated plates, which is one of the many elements of the structural performance of structures that are formed of composite materials. In particular, it is vital to take into mind the buckling phenomenon that occurs in such plates in order to ensure that the design is both efficient and dependable, as well as to ensure that the structural element may be used safely. The analysis of composite laminated plates is often more difficult than the analysis of homogeneous isotropic plates. This is because the anisotropic and coupled material behavior of composite laminated plates makes the analysis more challenging.

The members and structures that are made up of laminated composite material are often quite thin, and as a result, they are more prone to buckling. Buckling is a phenomena that poses a significant threat to structural components. This is due to the fact that the buckling of composite plates often takes place at a lower applied stress and results in significant deformations. Because of this, the research of buckling behavior in composite materials became the primary focus of attention.

The buckling of elastic structures and laminated plates is discussed in general in references such as [1] and [2]. These references provide general introductions to the subject. The curves and data that are

now accessible, on the other hand, are limited to idealized loads, namely uniaxial or biaxial homogeneous compression.

## 2. THE PAST WORK OF BUCKLING ANALYSIS

The quantity of research addressing comparable stability issues is examined by a diverse array of analytical techniques, which may be categorized as closed-form analytical, semi-analytical, or fully numerical approaches.

Exact closed-form solutions for the buckling issue of rectangular composite plates exist just for a restricted set of boundary conditions and lamination configurations. This encompasses cross-ply symmetric and angle-ply anti-symmetric rectangular laminates with a minimum of two opposite edges simply supported, as well as analogous plates with two opposite edges clamped yet permitted to deflect (i.e., guided clamp) or with one edge simply supported and the opposite edge featuring a guided clamp. The majority of the precise solutions examined in the treatises of Whitney, who formulated an accurate solution for the critical buckling of solid rectangular orthotropic plates with all corners simply supported, as well as those of Reddy, Leissa, and Kang. Bao et al. produced an exact solution for two simply supported edges and two clamped edges, whereas Robinson formulated an exact solution for the critical buckling stress of an orthotropic sandwich plate with all edges simply supported.

For all other configurations, for which only approximate findings are accessible, several semi-analytical and numerical methods have been devised. The Rayleigh-Ritz method, the finite strip method (FSM), the element-free Galerkin method (EFG), the differential quadrature technique, the moving least squares differential quadrature method, and the widely utilized finite element method (FEM) are the most prevalent techniques.

The Kantorovich method (KM) is a distinct and often beneficial semi-analytical technique that integrates a variational approach with closed-form answers and an iterative process. The technique presupposes a solution represented as a summation of products of functions oriented in one direction and functions oriented in the opposite direction. Subsequently, by assuming the function in a singular direction, the variation issue of the plate is simplified to a system of ordinary differential equations. In buckling analysis, the variation issue simplifies to an ordinary differential eigenvalue and eigenfunction problem. The solution to the resultant issue is approximate, and its precision is contingent upon the assumed functions in the initial direction. The Extended Kantorovich Method (EKM), introduced by Kerr, serves as the foundation for an iterative process whereby the solution derived in one route is used as the assumed functions in the subsequent direction. Upon reiterating this procedure several times, convergence is achieved. The singular phrase extended Kantorovich technique was used for the buckling analysis of rectangular plates by many researchers. Eienberger and Alexandrov [6] used the approach for the buckling analysis of isotropic plates exhibiting varying thickness. Shufrin and Eisenberger expanded the answer to thick plates with both constant and variable thickness by using first and higher-order shear deformation theories.

Ungbhakorn and Singhatanadgid expanded the solution for the buckling of symmetrically cross-ply laminated rectangular plates. The multi-term formulation of the extended Kantorovich method for basic samples of rectangular isotropic plates was introduced by Yuan and Jin [7]. This research shown that the supplementary words in the expansion may enhance the answer.

March and Smith [ 8] discovered an approximate solution applicable to all clamped edges. Chang et al. [9] devised an approximate solution for the buckling of rectangular orthotropic sandwich plates with two edges simply supported and two edges clamped, or with all edges clamped, using the March-Erickson approach and an energy strategy. Jiang et al. [10] formulated solutions for the local buckling of rectangular orthotropic hat-stiffened plates, with edges parallel to the stiffeners either simply supported or clamped, while the edges parallel to the stiffeners were free. Smith [11] provided solutions that bound the local buckling of hat-stiffened plates by treating the section between stiffeners as simply supported or clamped plates.

Numerous authors have used the finite element approach to accurately estimate in-plane stress distribution, which is then utilized to address the buckling issue. Cook has distinctly outlined a methodology for determining the buckling strength of plates by initially addressing the linear elastic problem for a reference load, followed by solving the eigenvalue problem to ascertain the smallest eigenvalue, which, when multiplied by the reference load, yields the critical buckling load of the structure. Yang et al. provided an exemplary assessment of the evolution of plate finite elements during the last 35 years.

A multitude of buckling studies for composite plates documented in the literature are often conducted concurrently with vibration analyses and are based on two-dimensional plate theories, which may be categorized as classical or shear deformable. Classical plate theories (CPT) neglect shear deformation effects, resulting in an overestimation of critical buckling loads for larger composite plates, as well as for thinner plates exhibiting more anisotropy. Most shear deformable plate theories are typically predicated on a displacement field assumption of five unknown displacement components. Three of these components align with those in CPT, while the extra components are multiplied by a specific function of the thickness coordinate and included into the displacement field of CPT to account for shear deformation effects. Considering these functions as linear and cubic forms results in the uniform or Mindlin shear deformable plate theory (USDPT) and parabolic shear deformable plate theories (PSDPT), respectively. Researchers also used several methodologies, including hyperbolic shear deformable plate theory (HSDPT) and trigonometric or sine functions shear deformable plate theory (TSDPT). Due to the inability of these shear deformation theories to fulfill continuity constraints across several layers of composite structures, zig-zag plate theories were presented by Di Sciuva and by Cho and Parmeter to account for interlaminar stress continuity. Recently, Karama et al. proposed a novel exponential function, termed exponential shear deformable plate theory (ESDPT), to characterize the displacement field of composite laminated structures. This function facilitates the representation of shear stress distribution throughout the thickness of the composite structures, and the authors compared their findings for both static and dynamic

problems of composite beams with the sine model.

In classical lamination theory, Jones, provided a closed-form solution for the buckling issue of cross-ply laminated plates with simply supported boundary conditions. For multi-layered plates under diverse boundary conditions that deviate from simply supported conditions at all four edges, the governing equations for the buckling of composite plates do not permit an exact solution, except in specific configurations of laminated plates. Consequently, diverse analytical and/or numerical methodologies are used by various scholars to address these sorts of challenges. Baharlou and Leissa used the Ritz approach using simple polynomials as displacement functions within classical theory to address the buckling issue of cross and angle-ply laminated plates under arbitrary boundary conditions and varying in-plane loads. Narita and Leissa also used the Ritz technique, assuming the displacement components as a double series of trigonometric functions, to address the buckling issue of usually symmetric laminated composite rectangular plates with simply supported boundary conditions along all edges. The researchers examined the essential buckling loads under five distinct loading conditions: uniaxial compression (UA – C), biaxial compression (BA – C), biaxial compression-tension (BA – CT), and both positive and negative shear loads.

Higher-order shear deformation theories may provide more precise inter-laminate stress distributions. The use of cubic variation of displacement eliminates the need for shear corrective displacement. To attain a dependable analysis and secure design, the recommendations and advancements of models using higher-order shear deformation theories have been evaluated. Lo et al. evaluated the seminal contributions in the area and developed a theory that incorporates the effects of transverse shear deformation, transverse strain, and the nonlinear distribution of in-plane displacements relative to the thickness coordinate. Third-order theories have been posited by Reddy, Librescu, Schmidt, Murty, Levinson, Seide, Murthy, Bhimaraddi, Mallikarjuna and Kant, as well as Kant and Pandya, among others. Innovative research and comprehensive reviews in the domain of closed-form solutions and finite element models are available in the works of Reddy, Mallikarjuna and Kant, Noor and Burton, Bert, Kant and Kommineni, and Reddy and Robbins, among others.

In the buckling analysis of cross-ply laminated plates with simply supported boundary conditions on two opposing edges and varying boundary conditions on the other edges, Khdeir used a parabolic shear deformation theory and utilized the state-space approach. Hadian and Nayfeh, based on the same theory and addressing the same sort of problem, need modifications to the approach owing to ill-conditioning issues, particularly for thin and moderately thick plates. Fares and Zenkour presented the buckling analyses of entirely simply supported cross-ply laminated plates, incorporating a non-homogeneity coefficient in the material stiffnesses across various plate theories. Matsunaga utilized a global higher-order plate theory in his analysis. Gilat et al. also examined a similar issue based on a global-local plate theory, wherein the displacement field comprises both global and local contributions, thereby allowing for the integration of continuity conditions and delamination effects into the formulation.

Numerous studies have been documented about the static and stability study of composite laminates using various conventional methodologies. Pagano devised a precise three-dimensional

elasticity solution for the static analysis of rectangular bi-directional composites and sandwich plates. Noor offered a solution for the stability of multi-layered composite plates based on three-dimensional elasticity theory by using the finite difference approach to solve the equations. Gu and Chattopadhyay provide 3-D elasticity solutions for the buckling of simply supported orthotropic composite plates. The reduction of the issue from three dimensions (3-D) to two dimensions facilitates a more effective computer study of composite plate structures, hence drawing significant focus on displacement-based theories and their associated finite element models.

The bifurcation buckling of laminated constructions has been examined by several researchers without accounting for flatness prior to buckling. This aspect was first elucidated for laminated composite plates under certain boundary circumstances and lamina configurations by Leissa. Qatu and Leissa used this finding to ascertain the authentic buckling behavior of composite plates.

Elastic bifurcation of plates has been thoroughly examined and widely recorded in standard manuals, research monographs, and journal articles.

### 3. CONCLUSIONS

In this presentation, a comprehensive bibliography and literature study on the topic of buckling of composite laminated plates were provided and completely explored. Using this method, exact, analytical, and semi-analytical solutions in the buckling of laminates were investigated.

A variety of elements, such as the size of the plate and the lamination scheme, are included in the boundary conditions. In the process of analyzing buckling, the development of plate theories, beginning with the classical plate theory and progressing through first order shear deformation and finally reaching higher order shear deformation theories, was taken into consideration. Through research, it was shown that higher order shear deformation theories are capable of producing inter-laminar stresses that are more precise, and they also eliminate the need for shear corrective displacement.

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