Bramgupta:- A Poineer of Indian Mathematics

Dr. Jai Nandan Singh
HOD- Mathematics
Koshi College Khagaria
T.M.B.U. Bhagalpur

ABSTRACT-

In this paper we explores the significant Contributions of Brahmagupta , an ancient Indian mathematician whose work in the 7th Century CE influenced mathematics, astronomy and Various Scientific fields. His main key focus areas include pioneering work on zero, rules of arithmetic involving negative numbers, Solutions to quadratic and indeterminate equations and Contributions of geometry, particularly his formula for the area of cyclic quadrilaterals. Here we also go to show the global influence of Brahmagupta's global influence work, showing how his insights laid the foundation for modern mathematical thought and for Coming generation.

According to Tobias Danzing “In the history of culture the discovery of Zero will aways Stand out as one of the greatest single achievement of the human race."

Key words :- Ancient, Astronomy, Scientific, Felds, Poineer, Indenterminate, Equation.

Introduction :-

Brahmagupta was born in the city of Bhinmal,(Ujjain), Rajasthan, Hindustan during the region of King Vyaghramukha of the chaulukya dynasty. He was the sun of Jishnu and the grandson of Prabhakara. Brahmagupta’s exact birth and dates are uncertain, but based on astronomical observation in his Brahmsphutasiddhanta, it is estimated that he lived between 598 and 668 AD. He was a great Indian mathematician and astronomer who wrote several books, at the age of 30, in the year 628 A.D. He wrote a treatise called Brahmasphutasiddhanta, it was one of his initial works. It is speculated that Brahmagupta revised the text of Siddhanta that he received; nevertheless, he also brought a great deal of new material into it.

A large part of this treatise concerns astronomy, but it also includes mathematics, trigonometry, algorithms and algebra. While further developing the already known notion of the placeholder in numbers, Brahmagupta actually realized that mathematics needs a new number. Consequently, in his work he introduced one of the fundamental discoveries in mathematics the concept of the number zero. Brahmagupta called the new number **“shunya”**, which in Sanskrit means "void" or "empty" or “Null” or “Zero”

Brahmagupta was the first mathematician who conceived the concept of zero, and defined its arithmetical properties, respectfully he is called the father of zero. He argued that shunya (zero) is a number that represents nothing, and stated "shunya is actually the result of subtracting a number from itself", a concept which now a days. Every one understands, in his time however it was a completely new and current idea.

Brahmagupta also criticized contemporary Indian astronomers and mathematicians. The core of the dispute was related to the differing views and concepts; in fact he devoted several chapters of Brahmasphutasiddhanta to criticize their mathematical concepts. After Arabs conquered a part of India, Brahmagupta's works were translated by the astronomer Muhammad alFazari into Arabic, thus promoting the advance and implementation of the Indian base ten number system, according to Hoffman a great astronomer and mathematician of Arabic (Islamic Reign).

Major Contributions :-

1. History of ZERO

1.1. The place holder Concept

Already the ancient mathematicians of Babylonia, Greece and China recognized that a placeholder is necessary in order for the numbers to display and hold the correct values. The placeholder concept was invented independently in a number of civilizations around the world, says Dr. Annette van der Hoek, the lead researcher at the Zero Project.

The Sumerians were the first people in the world to develop a counting system, they developed their counting system around 4,000 to 5,000 years ago. The Sumerian counting system, similarly to the modern decimal system, was positional; this means that the value of a symbol depended on its position in the sequence of other symbols.

Around 300 B.C. the Sumerian system passed from the Akkadian Empire to the Babylonians who were using base 60 counting system. It was the Babylonians, Harvard mathematics professor Robert Kaplan maintains, that implemented a symbol that obviously was a placeholder. The placeholder facilitated clear discernment of 10 from 100 for example. Initially, the Babylonians left an empty space in their cuneiform number, but when that became confusing, they implemented in its place double angled wedges. Babylonians however, never developed the idea of a zero as a number. The Mayans, independently invented zero as a placeholder around 350 A.D. implementing it in their calendar system. Kaplan says that despite being highly skilled mathematicians, the Mayans never used zero in equations. The Mayan invention of zero is the "most striking example of the zero being devised wholly from scratch".

In 1891 archeologist A. Leclere discovered an inscription in Cambodia, containing the "dot" as a placeholder for zero. This inscription, later classified as K-127 by G. Coedes, has been lost for decades, following the Khmer Rouge destruction of cultural artifacts. Later the stone containing the inscription has been re-discovered by A. Aczel. "Archeologists date this phrase to 687 AD, in Pre-Angkor Cambodia has many inscriptions with the zero symbol, but this one K-127 is the oldest one" Chea Socheat. It is the oldest record of reference to zero in Cambodia. A symbolic placeholder however, cannot be equated with zero itself. Because it lacks the most important qualities of zero precise arithmetical properties.

The Bakhshali manuscript is an early ancient mathematical document discovered in a field in 1881 by a farmer, near the village of Bakhshali in the vicinity of Peshawar, at that time in Bharat. The manuscript is written in ink on a birch bark, which was usual for manuscripts in North-Western India. It displays some Sanskrit numerals, in which in the place of zero is placed a small dot. It is a compendium of rules and examples and presents verified solutions. The content of the manuscript covers square root computations to extraordinary accuracy, fractions, quadratic equations, simultaneous linear equations, indeterminable equations of the second degree and many other problems.

The fragments comprising its pages, were once part of a more extensive document. With a considerable part of it missing and the uncertainty of the correct order of the pages, we may only try to guess its significance. Attempts to assess the manuscript were published by G. Kaye of the Indian Government in 1927 and Takao Hayashi in 1995. Hayashi attempted to order the pages and translate them, his translation is considered superior to the 1927 translation by Kave. The manuscript had been donated in 1902 to the Oxford University Library. In 2017 researchers at Oxford University carried out a comprehensive research of the Bakhshali manuscript, aiming to carbon date the document. The manuscript has been dated to be from ca 300 to 400 A.D., this carbon dating however resulted in a strong criticism by a group of researchers cf. Plofker et all . Independently of that, a much earlier thorough research done by Channabasappa as well as Data based on the results contained in the Bakhshali manuscript, yielded the approximate origin of the manuscript to be from ca 200 to 400 A.D. Channabasappa derived from the Bakhshali manuscript an iterative method to calculate square roots, which is significantly faster than the Newton's method.

According to Casselman the Bakhshali manuscript does not tell us much new, what has not been seen in other Indian mathematical manuscripts. It does however present "Extraordinarily accurate approximations to the square roots of integers that are not perfect squares... My own personal belief is that the original work from which the Bakhshali manuscript originates was very close to the invention of the full decimal place system of arithmetic"."

1.2. The birth and advance of zero

The Indian Hindu culture had a positional number system implementing the base ten. In these early counting systems, the placeholder has not been considered a number with its own properties. Comprehension of zero's concept significance and its properties, developed first during the seventh century A.D. In India. In the year 628 A.D. Brahmagupta wrote a treatise called Brahmasphutasid-dhanta (translated as "The opening of the Universe"). Brahmagupta actually realized, that mathematics needs a new number. Consequently, in the treatise he introduced one of the fundamental discoveries in mathematics the concept of the number zero. Brahmagupta called the new number súnya, which in Sanskrit means "void" or "empty".

Hindu mathematicians Aryabhata born in 476 A.D. and Brahmagupta born in 598 A.D. are believed to be the first mathematicians who formally described the modern decimal number system.

"The creation of zero as a number in its own right, which evolved from the placeholder dot symbol found in the Bakhshali manuscript, was one of the greatest breakthroughs in the history of mathematics. We know that it was as early as the 3rd Century that mathematicians in India planted the seed of the idea that would later become so fundamental to the modern World", said Marcus du Sautoy, Professor of Mathematics at University of Oxford, the lead researcher on Mathematics.

The oldest written record of zero as a number in its own right is located in the Chaturbhuj Temple in Gwalior, India where a 9th century carving on the wall displays the number 270, according to M. Ward.

2. Comprehension of Zero's significance :-

2.1. Comprehension of zero's significance and its properties spread also beyond In dia. The concept of zero initially made its way to China and then back to the Middle East, where it was taken up by the mathematician Mohammed ibn-Musa al-Khowarizmi around 773, becoming part of the Arabic number system based upon the Indian system, according to Hoffman .

2.2. It was Al-Khowarizmi who first synthesized Indian arithmetic and showed how zero could function in algebraic equations, and by the ninth century the zero had Tentered the Arabic numeral system in a form resembling the oval shape we use today. The Arabs called this circle "sifr," or "empty"; the method for multiplying and dividing numbers, which now a days is known as an algorithm, is a derived from the title of the latin version of his book.

2.3. Zero is undoubtedly one of the greatest achievements in mathematics. The actual term zero was coined in Italy. In the decimal number system over R, zero separates the negative and the positive numbers, itself being neither negative nor positive. Zero facilitated the rise of the modern mathematics as we know it. "So common place has zero become, that few if any, realize its astounding role in the lives of every single person in the world" said Peter Gobets, the secretary of the ZerOrig India Foundation, or the Zero Project.

2.4. Further he says that "the Indian concept of zero, widely seen as one of the greatest innovations in human history, is the cornerstone of modern mathematics and physics, plus the spin-off technology". Hence, it is behind every modern achievement, as it lies at the very foundation of the modern science and technology. It is omnipresent today, its applications stretch from mathematics to engineering, in fact zero permeates our everyday lives.

3. The banning of zero:-

The concept of zero spread to Europe ca 1200, popularized in the work of Leonardo Fibonacci of Pisa Italy Liber Abaci, Matson, Fibonacci, used it to evaluate equations. Fibonacci's equations involving zero were favored among merchants, who used it to balance their books.

Soon afterward however, zero was banned in Europe because, as the proponents of such a move maintained, zero promotes the possibility of fraud. They said that zero can easily be altered to become 9, nothing stops one from adding a few zeroes at the end of a number etc. As a result, zero and the Arabic numerals were banned in 1299. It took approximately 200 years before the Arabic numerals and zero were finally accepted in Europe, according to Fry .

After that, the concept of "nothing" has continued to play a vital role in the development of everything from science and engineering to technology. According to Prof. Brian Rotman "Between the tenth and the thirteenth century the sign (Zero) stayed within the confines of Arab culture, resisted by Christian Europe, and dismissed by those whose function it was to handle numbers as an incomprehensible and unnecessary symbol... But, as will be obvious by now, the mathematical infinite was the fruit of the mathematical nothing: “It is only by virtue of zero that infinity comes to be significant in mathematics", according to Rotman. Although zero and nothing point to a state of absence they are not equivalent. Therefore the question arises: what is the difference between zero and nothing.

• Zero is a number which has very precise arithmetical properties, it is the result of arithmetical and counting process. Nothing is just an abstract concept which has no arithmetical properties, it is uncountable it either is true or false.

• Zero is the only number which is neither positive nor negative at the same time, it has an exact numerical value and position on the number line. Nothing neither has a position on the number line nor exact arithmetical value.

4. Properties of ZERO :-

Brahmagupta's ten rules for zero, according to Bentley:-

(a) A debt minus zero is a debt.
(b) A fortune minus zero is a fortune.
(c) Zero minus zero is zero.
(d) A debt subtracted from zero is a fortune.
(e) A fortune subtracted from zero is a debt.
(f) The product of zero and a debt or fortune is zero.
(g) The product of zero and zero is zero.
(h) Positive or negative numbers, when divided by zero result in a fraction with the zero in the denominator.
(i) Zero divided by a negative or a positive number is either zero, or is ex- pressed as a fraction with zero as numerator and the finite quantity in the denominator.
(j) Zero divided by zero is zero.

5. Negative numbers :-

In the Brahmasphutasiddhanta, Brahmagupta addressed operations involving negative numbers, distinguishing between positive (known as "fortunes") and negative numbers (known as "debts"). He developed rules for operations with negatives, a concept that would not be formally embraced in Western mathematics until centuries later.

6. Quadratic Equations and Algebra :-

6.1. Algebra

Brahmagupta gave the solution of the general linear equation in chapter eighteen of Brahmasphuṭasiddhanta , The difference between rupas, when inverted and divided by the difference of the coefficients of the, is the unknown in the equation. The rupas are below that from which the square and the unknown are to be subtracted, which is a solution for the equation $bx + c = dx + e$ where rupas refers to the constants c and e. The solution given is equivalent to $x=e-c/b-d$.

He further gave two equivalent solutions to the general quadratic equation Diminish by the middle number the square-root of the rupas multiplied by four times the square and increased by the square of the middle number divide the remainder by twice the square. The result is the middle number. Whatever is the square-

root of the rupas multiplied by the square and increased by the square of half the unknown, diminished that by half the unknown and divide the remainder by its square. The result is the unknown.
which are, respectively, solutions for the equation ax + bx = c equivalent to, and He went on to solve systems of simultaneous indeterminate equations stating that the desired variable must first be isolated, and then the equation must be divided by the desired variable's coefficient. In particular, he recommended using "the pulverizer" to solve equations with multiple unknowns.

Subtract the colors different from the first color. The remainder divided by the first color's coefficient is the measure of the first. Terms two by two are considered when reduced to similar divisors, and so on repeatedly. If there are many colors, the pulverizer is to be used.

Like the algebra of Diophantus, the algebra of Brahmagupta was syncopated. Addition was indicated by placing the numbers side by side, subtraction by placing a dot over the subtrahend, and division by placing the divisor below the dividend, similar to our notation but without the bar. Multiplication, evolution, and unknown quantities were represented by abbreviations of appropriate terms. The extent of Greek influence on this syncopation, if any, is not known and it is possible that both Greek and Indian syncopation may be derived from a common Babylonian source.

6.2. Arithematic:-

The four fundamental operations addition, subtraction, multiplication, and division were known to many cultures before Brahmagupta. This current system is based on the Hindu- Arabic numeral system and first appeared in the Brahmasphutasiddhānta. Brahmagupta describes multiplication in the following way:

The multiplicand is repeated like a string for cattle, as often as there are integrant portions in the multiplier and is repeatedly multiplied by them and the products are added together. It is multiplication. Or the multiplicand is repeated as many times as there are component parts in the multiplier. Indian arithmetic was known in Medieval Europe as modus Indorum meaning "method of the Indians". In the Brahmasphuṭasiddhanta, four methods for multiplication were described, including gomūtrikā, which is said to be close to the present day methods. In the beginning of chapter twelve of his Brahmasphuṭasiddhānta, entitled "Calculation", he also details operations on fractions. The reader is expected to know the basic arithmetic operations as far as taking the square root, although he explains how to find the cube and cube-root of an integer and later gives rules facilitating the computation of squares and square roots. He then gives rules for dealing with five types of combinations of
fractions: a/c + b/c; a/c × b/d; a/1 + b/d; a/c + b/d x a/c = a(d + b)/cd; and a/c - b/d x a/c = a(d-b)/cd.

6.3. Squares and Cubes :-

Brahmagupta then goes on to give the sum of the squares and cubes of the first integers. The sum of the squares is that sum multiplied by twice the number of steps increased by one and divided by three. The sum of the cubes is the square of that sum Piles of these with identical balls can also be computed,
Here Brahmagupta found the result in terms of the sum of the first integers, rather than in terms of it as is the modern practice,
He gives the sum of the squares of the first / natural numbers as n (n+1)(2n+1)/6 and the sum of the cubes of the first n natural numbers as $\{n(n+1)/2\}^2$

6.4. Diophantine analysis :-

In chapter twelve of his Brähmasphutasiddhänta, Brahmagupta provides a formula useful for generating Pythagorean triples:-
The height of a mountain multiplied by a given multiplier is the distance to a city; it is not erased. When it is divided by the multiplier increased by two it is the leap of one of the two who make the same journey,
Or, in other words, if d = mx / x + 2 then a traveller who "leaps" vertically upwards a distance d from the top of a mountain of height , and then travels in a straight line to a city at a horizontal distance mx from the base of the mountain, travels the same distance as one who descends vertically down the mountain and then travels along the horizontal to the city Stated geometrically, this says that if a right-angled triangle has a base of length a = mx and altitude of length b = m + d then the length, c, of its hypotenuse is given by c = m(1 + x) – d. And, indeed, elementary algebraic manipulation shows that $a^2 + b^2 = c^2$, whenever d has the value stated. Also, if m and x are rational, so are d, a, b and c. A Pythagorean triple can therefore be obtained from a, b and c by multiplying each of them by the least common multiple of their denominators.

7.Pell's equation :-

Brahmagupta went on to give a recurrence relation for generating solutions to certain instances of Diophantine equations of the second degree such as N $x^2 + 1 = y^2$ called Pell's equation by using the Euclidean algorithm. The Euclidean algorithm was known to him as the "pulverizer" since it breaks numbers down into ever smaller pieces, The nature of squares twice the square-root of a given square by a multiplier and increased or diminished by an arbitrary number. The product of the first pair, multiplied by the multiplier, with the product of the last pair, is the last computed. The sum of the thunderbolt products is the first. The additive is equal to the product of the additives. The two square-roots, divided by the additive or the subtractive, are the additive rupas, The key to his solution was the identity,

which is a generalisation of an identity that was discovered by Diophantus, In some of the verses before verse 40, Brahmagupta gives constructions of various figures with arbitrary sides. He essentially manipulated right triangles to produce isosceles triangles, scalene triangles, rectangles, isosceles trapezoids, isosceles trapezoids with three equal sides, and a scalene cyclic quadrilateral.

After giving the value of pi, he deals with the geometry of plane figures and solids, such as finding volumes and surface areas or empty spaces dug out of solids. He finds the volume of rectangular prisms, pyramids, and the frustum of a square pyramid. He further finds the average depth of a series of pits. For the volume of a frustum of a pyramid, He gives the "pragmatic" value as the depth times the square of the mean of the edges of the top and bottom faces, and he gives the "superficial" volume as the depth times their mean area.

8. Trigonometry :-

In Brahmasphuṭasiddhanta, entitled Planetary True Longitudes, Brahmagupta presents a sine table: The sines: The Progenitors, twins; Ursa Major, twins, the Vedas; the gods, fires, six; flavors, dice, the gods; the moon, five, the sky, the moon, arrows, suns Here Brahmagupta uses names of objects to represent the digits of place- value numerals, as was common with numerical data in Sanskrit treatises. Progenitors represents the 14 Progenitors "Manu" in Indian cosmology or 14, "twins" means 2, "Ursa Major" represents the seven stars of Ursa Major or 7. "Vedas" refers to the 4 Vedas or 4, dice represents the number of sides of the traditional die or 6, and so on. This information can be translated into the list of sines, 214, 427, 638, 846, 1051, 1251, 1446, 1635, 1817, 1991, 2156, 2312, 2459, 2594, 2719, 2832, 2933, 3021, 3096, 3159, 3207, 3242, 3263, and 3270, with the radius being 3270 this numbers represent for Interpolation formula.

Main article: Brahmagupta's interpolation formula:-

In 665 Brahmagupta devised and used a special case of the Newton-Stirling interpolation formula of the second-order to interpolate new values of the sine function from other values already tabulated. The formula gives an estimate for the value of a function fat a value a + x/ of its argument when its value is already knownat ah, a and a + h.

9. Concept of Gravity :-

Brahmagupta in 628 first described gravity as an attractive force, using the term "gurutvakarsanam (गुरुत्वाकर्षणम)" to describe it: for some rational numbers u, v, and w.

Brahmagupta's theorem

Brahmagupta's theorem states that AF = FD. The square-root of the sum of the two products of the sides and opposite sides of a non-unequal quadrilateral is the diagonal. The square of the diagonal is diminished by the square of half the sum of the base and the top; the square-root is the perpendicular.

So, in a "non-unequal" cyclic quadrilateral the length of each diagonal is √pr + qs.

He continues to give formulas for the lengths and areas of geometric figures, such as the circumradius of an isosceles trapezoid and a scalene quadrilateral, and the lengths of diagonals in a scalene cyclic quadrilateral. This leads up to Brahmagupta's famous theorem,

10. Pi

He gives values of π. The diameter and the square of the radius multiplied by 3 are the practical circumference and the area . The accurate values are the square-roots from the squares of those two multiplied by ten, So Brahmagupta uses 3 as a "practical" value of π, and as an "accurate" value of π, with an error less than 1%.

11.Astronomy

Brahmagupta directed a great deal of criticism towards the work of rival astronomers, and his Brähmasphutasiddhānta displays one of the earliest schisms among Indian mathematicians. The division was primarily about the application of mathematics to the physical world, rather than about the mathematics itself. In Brahmagupta's case, the disagreements stemmed largely from the choice of astronomical parameters and theories. Critiques of rival theories appear throughout the first ten astronomical chapters and the eleventh chapter is entirely devoted to criticism of these theories, although no criticisms appear in the twelfth and eighteenth chapters.

In chapter seven of his Brähmasphuţasiddhanta, entitled Lunar Crescent, Brahmagupta rebuts the idea that the Moon is farther from the Earth than the Sun, accline He does this by explaining the illumination of the Moon by the Sun.

11.1 If the moon were above the sun, how would the power of waxing and waning, etc., be produced from calculation of the longitude of the moon. The near half would always be bright.

11.2. In the same way that the half seen by the sun of a pot standing in sunlight is bright, and the unseen half dark, so is of the moon beneath the sun.

11.3 The brightness is increased in the direction of the sun. At the end of a bright i.e. waxing half-month, the near half is bright and the far half dark. Hence, the elevation from calculation.

He explains that since the Moon is closer to the Earth than the Sun, the degree of the illuminated part of the Moon depends on the relative positions of the Sun and the Moon, and this can be computed from the size of the angle between the two bodies.

Further work exploring the longitudes of the planets, diurnal rotation, lunar and solar eclipses, risings and settings, the moon's crescent and conjunctions of the planets, are discussed in his treatise Khandakhadyaka.

Conclusion :-

Brahmagupta's works encapsulate the achievements of ancient Indian mathematics, with contributions that shaped arithmetic, algebra, and geometry. His groundbreaking understanding of zero, negative numbers, quadratic solutions, and geometry has transcended centuries, making Brahmagupta not only a foundational figure in Indian mathematics but also a global pioneer. His influence remains woven into the fabric of modern mathematical and scientific thought, testifying to the enduring legacy of ancient Indian intellectual advancements.

References

1.Joseph, G.G. (2000). The Crest of the Peacock: Non-European Roots of Mathematics
2.Hayashi, T. (1995). "Brahmagupta's Mathematics" in Indian Journal of History of Science
3.Ifrah, G. (2000). The Universal History of Numbers
4.Medicine in Non-Western Cultures, Springer, pp. 162-163, ISBN 978-1-4020-4559-2
5.O'Leary, De Lacy (2001) [first published 1948], How Greek Science
6.Passed to the Arabs (2nd ed.), Goodword Books, ISBN 8187570245
7.Plofker, Kim (2007), "Mathematics in India", in Victor Katz(ed.), The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook, Princeton University Press, ISBN 978-0-691- 11485-9
8.Stillwell, John (2009), Mathematics and its History (Second ed.), Springer Science + Business Media Inc., ISBN 0-387-95336-1
9.Hockey, Thomas, ed. (2007), "Brahmagupta", Biographical Encyclopedia of Astronomers, Springer Science & Business Media p. 165, ISBN 978-0387304007.