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On Fuzzy Pseudo Lindelöf Spaces

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Abstract: In this paper, the concept of fuzzy pseudo Lindelof spaces is introduced. Several characterizations of fuzzy pseudo Lindelof spaces are given. The conditions under which fuzzy pseudo Lindelof spaces are obtained.

Key Words : Fuzzy dense set, Fuzzy nowhere dense set, Fuzzy first category set, Fuzzy simply open set, Fuzzy resolvable set, Fuzzy pseudo open set, Fuzzy hyper connected space, Fuzzy strongly irresolvable space, Fuzzy sub maximal space.

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I. INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by **L.A. Zadeh[14]** in 1965. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of mathematics in 1968 **C.L.Chang[5],** applied basic concepts of general topology to fuzzy sets and introduced fuzzy topology. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. In the recent years, there has been a growing trend among many fuzzy topologists to introduce and study different forms of fuzzy sets and a considerable amount of research has been done on many types of fuzzy continuity in fuzzy topology.

In 1899, **Rene Louis Baire [2]** introduced the concepts of first and second category sets in his doctoral thesis. By means of first category sets, the notion of pseudo open sets was introduced and studied by **A.Neubrunnova [6]** in classical topology. The purpose of this paper is to introduce and study fuzzy pseudo open sets in fuzzy topological spaces. A new class of spaces, called fuzzy pseudo Lindelof space between fuzzy topological spaces, is introduced and studied. It is observed that the fuzzy pseudo open sets in fuzzy hyper connected spaces are fuzzy simply open sets and fuzzy resolvable sets. The conditions under which fuzzy hyper connected spaces become fuzzy Baire spaces and fuzzy volterra spaces, are also obtained by means of fuzzy pseudo open sets. Several examples are given to illustrate the concepts introduced in this paper.

II. PRELIMINARIES

In order to make the exposition self- contained, some basic notions and results used in the sequel are given. In this work (X,T) or simply by X, we will denote a fuzzy topological space due to **Chang** (1968). Let X be a non- empty set and I, the unit interval [0,1]. A fuzzy set λ in X is a function from X into I.The null set 0 is the function from X into I which assumes only the value 0 and the whole fuzzy set 1 is the function from X into I takes 1 only.

Definition 2.1[5]: Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T).The interior and the closure of λ defined as follows

(i) $Int(\lambda) = \vee \{ \mu / \mu \leq \lambda, \ \mu \in T\}$
(ii) $Cl(\lambda) = \wedge \{ \mu / \lambda \leq \mu , 1-\mu \in T\}$

Lemma 2.1[1]: For a fuzzy topological space X,

(i) $1 - Int(\lambda) = cl(1 - \lambda)$
(ii) $1 - Cl(\lambda) = int(1 - \lambda)$

Definition 2.2[8]: A fuzzy set λ in a fuzzy topological space (X,T), is called fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $cl(\lambda) = 1$, in (X,T).

Definition 2.3[8]: A fuzzy set λ in a fuzzy topological space (X,T), is called fuzzy nowhere dense if there exists no non –zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $intcl(\lambda) = 0$, in (X,T).

Definition 2.4[3]: A fuzzy topological space (X,T) is called fuzzy first category set if $\lambda = \vee_{\alpha=1}^{\infty}(\lambda_\alpha)$,where (λ_α)'s are fuzzy nowhere dense sets in (X,T).Any other fuzzy set in (X,T) is said to be of fuzzy second category.

Definition 2.5[11]: Let λ be a fuzzy first category set in (X,T). Then $1 - \lambda$ is called a fuzzy residual set in (X, T).

Definition 2.6[3]: A fuzzy topological space (X,T) is called fuzzy first category if $1_x = \vee_{\alpha=1}^{\infty}(\lambda_\alpha)$,where (λ_α)'s are fuzzy nowhere dense sets in (X,T). A fuzzy topological space which is not of fuzzy first category is said to be of fuzzy second category.

Definition 2.7[10]: A fuzzy topological space (X,T) is called a fuzzy strongly irresolvable space if for every fuzzy dense set λ in (X,T), clint(λ) = 1 in (X,T).

Definition 2.8[7]: The fuzzy boundary of a fuzzy set λ in a fuzzy topological space (X,T) is defined as Bd(λ) = cl (λ) ∧ cl(1− λ)

Definition 2.9[12]: A fuzzy set λ in a fuzzy topological space (X,T), is called a fuzzy simply open set if Bd(λ) is a fuzzy nowhere dense set in (X,T).That is, λ is a fuzzy simply open set in (X,T) if cl (λ) ∧ cl (1− λ), is a fuzzy nowhere dense set in (X,T).

Definition 2.10[13]: A fuzzy set λ in a fuzzy topological space (X,T), is called a fuzzy simply* open set if λ = μ∨δ, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in (X,T) and 1− λ is called a fuzzy simply* closed set in (X,T).

Definition 2.11[3]: A fuzzy topological space (X,T) is called fuzzy submaximal space if for each fuzzy set λ in (X,T) such that cl(λ) = 1,then λ ϵ T in (X,T).

Definition 2.12[11]: A fuzzy topological space (X,T) is called a fuzzy Baire space if int($\vee_{\alpha=1}^{\infty}(\lambda_\alpha)$) = 0,where ($\lambda_\alpha$)'s are fuzzy nowhere dense sets in (X,T).

Definition 2.13[9]: A fuzzy topological space (X,T) is said to be fuzzy Lindelöf if for every fuzzy open cover of X has a countable sub cover. That is for every fuzzy open cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of X, there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ such that $\vee_{n \in N}$ $\{\lambda_{\alpha_n}\}$= 1.

Definition 2.14[4]: A fuzzy topological space (X, T) is said to be fuzzy hyperconnected, if every non-null fuzzy open subset of (X,T) is fuzzy dense set in (X,T). That is., a fuzzy topological space (X, T) is hyperconnected if cl(μ_i) =0, for all μ_iϵT.

Definition 2.15[13]: A fuzzy topological space (X,T) is said to be fuzzy simply Lindelöf if for every fuzzy simply open cover of X has a countable subcover. That is, (X,T) is a fuzzy simply Lindelof space if $\vee_{\alpha \in \Delta}\{\lambda_\alpha\}$=1, where intcl($\{\lambda_\alpha\}$) = 0 in (X,T), then $\vee_{n \in N}$ $\{\lambda_{\alpha_n}\}$= 1 in (X,T).

Definition 2.16[12]: Let (X,T) be a fuzzy topological space. A fuzzy set λ defined on X is called a fuzzy pseudo open set in (X,T) if λ = μ∨δ, where μ is a non-zero fuzzy open set in (X,T) and δ is a fuzzy first category set in (X,T) and 1- λ is called a fuzzy pseudo closed set in (X,T).

Theorem 2.1[13]: If λ is a fuzzy pseudo open set in a fuzzy hyperconnected space (X,T),then λ is a fuzzy simply open set in (X,T).

Theorem 2.2[12]: If λ is a fuzzy pseudo open set in a fuzzy topological space (X,T), then 1 – λ = β ∧ γ, where β is a fuzzy closed set in (X,T) and γ is a fuzzy residual set in (X,T).

Theorem 2.3[12]: If λ is a fuzzy pseudo open set in a fuzzy hyper connected space (X,T), then λ is a fuzzy resolvable in (X,T).

Theorem 2.4[11]: If int $\vee_{\alpha \in \Delta}$ $\{\lambda_\alpha\}$ = 0, where $\{\lambda_\alpha\}$'s are fuzzy resolvable sets in a fuzzy topological space (X,T), then (X,T) is a fuzzy Baire space.

Theorem 2.5[13]: If (X,T) is a fuzzy simply Lindelof spaces, then (X,T) is a fuzzy second category space.

III. ON FUZZY PSEUDO LINDELÖF SPACES

Definition 3.1: A collection $\{\lambda_\alpha : \alpha \in \Delta\}$ of fuzzy pseudo open sets of a fuzzy topological space (X,T) is said to be a fuzzy pseudo open cover of X if $\vee_{\alpha \in \Delta}$ $\{\lambda_\alpha\}$ = 1, in (X,T).

Definition 3.2: A fuzzy topological space (X,T) is said to be fuzzy pseudo Lindelöf if each cover of X by fuzzy pseudo open sets has a countable sub cover. That is (X,T) is a fuzzy pseudo Lindelof space if $\vee_{\alpha \in \Delta}$ $\{\lambda_\alpha\}$ = 1 where $\lambda_\alpha = (\mu_\alpha \vee \delta_\alpha)$, in which $\{\mu_\alpha\}$'s are non zero fuzzy open sets in (X,T) and $\{\delta_\alpha\}$'s are fuzzy first category sets in (X,T), then $\vee_{n \in N}$ $\{\lambda_{\alpha_n}\}$= 1, in (X,T).

Example 3.1: Let X = {a,b,c}.The fuzzy sets α, β, δ and λ are defined on X as Follows:

α : X → [0,1] is defined as α(a) = 0.5 ; α(b) = 1; α(c)=0.4;

β : X → [0,1] is defined as β(a) = 1; β(b) = 0.6; β(c)=1;

δ : X → [0,1] is defined as δ(a) = 0.6; δ(b) = 0.4; δ(c)=1;

λ : X → [0,1] is defined as λ(a) = 0.5; λ(b) = 1; λ(c)=0.6;

Then T = {0.α, β, δ, α∨δ, α∧β, α∧δ, δ∨[α∧β},1} is a fuzzy topology on X. On computation, one can find 1- α,1-β, 1- δ, 1- (α∨δ), 1 – [δ∨(α∧β)], are fuzzy nowhere dense sets in (X,T).Now 1- (α∧ δ) = (1- α)∨ (1- β)∨(1- δ)∨ [1 – (α∨δ)] ∨ {1- [δ∨(α∧β)]} shows that 1-(α∧δ) is a fuzzy first category set in (X,T). On computation are can see that fuzzy pseudo open sets in (X,T) are λ, β, δ∨(α ∧ β), 1- (α ∧ δ). Now λ ∨β ∨ [δ ∨ (α∧β)] ∨ [1- (α∧δ)] = 1, λ∨ β = 1, in (X,T).Shows that the fuzzy pseudo open cover {λ, β, 1- α ∧ δ, δ∨ (α ∧ β)} of (X,T) has a countable sub cover {λ, β} in (X,T) and hence (X,T) is a fuzzy pseudo Lindelof space.

Proposition 3.1: If ($\{\lambda_\alpha\}_{\alpha \in \Delta}$)' s are fuzzy pseudo open sets in a fuzzy pseudo Lindelof space (X,T), such that, $\vee_{\alpha \in \Delta}\{\lambda_\alpha\}$ = 1, then $\vee_{n \in N}$ $\{\lambda_{\alpha_n}\}$= 1, where int $(\lambda_{\alpha_n}) \neq 0$, in (X,T).

Proof: Let (X,T) be a fuzzy pseudo Lindelof space. Then, for the cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of X by fuzzy pseudo open sets in (X,T), there exists a countable sub cover $\{\lambda_{\alpha_n}\}_{n \in N}$ of X by fuzzy pseudo open sets. That is, $\vee_{n\in N}\{\lambda_{\alpha_n}\}$= 1, in (X,T). Since $\{\lambda_{\alpha_n}\}$'s are fuzzy pseudo open sets in (X,T), $\lambda_{\alpha_n} = (\mu_{\alpha_n} \vee \delta_{\alpha_n})$, where $\{\mu_{\alpha_n}\}$'s are non zero fuzzy open sets and $\{\delta_{\alpha_n}\}$'s are fuzzy first category sets in (X,T). Now, int(λ_{α_n}) = int $(\mu_{\alpha_n} \vee \delta_{\alpha_n}) \geq$ int $(\mu_{\alpha_n}) \vee$ int $(\delta_{\alpha_n}) = \mu_{\alpha_n} \neq 0$ and Hence int$(\lambda_{\alpha_n}) \neq 0$. Thus $\vee_{n \in N}\{\lambda_{\alpha_n}\}$= 1, where int $(\lambda_{\alpha_n}) \neq 0$, in (X,T).

Proposition 3.2: If $\wedge_{\alpha \in \Delta}$ $\{\mu_\alpha\}$ = 0 ,where $\{\mu_\alpha\}$'s are fuzzy pseudo closed sets in a fuzzy pseudo Lindelof space (X,T), then there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $\wedge_{n \in N}$ $\{\mu_{\alpha_n}\}$= 0 in (X,T).

Proof: Suppose that $\wedge_{\alpha \in \Delta}\{\mu_\alpha\}$ = 0 , where $\{\mu_\alpha\}$'s are fuzzy pseudo closed sets in (X,T). Now 1− $\wedge_{\alpha \in \Delta}$ $\{\mu_\alpha\}$ = 1, implies that $\vee_{\alpha \in \Delta}$ $\{1- \mu_\alpha\}$ = 1, in (X,T). Since (X,T) is a fuzzy pseudo Lindelof space and $\{1- \mu_\alpha\}$'s are fuzzy pseudo open sets in (X,T), there exists a countable sub cover$\{1- \mu_{\alpha_n}\}_{n \in N}$ of X by fuzzy pseudo open sets in (X,T). That is, there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $\vee_{n \in N}\{1- \mu_{\alpha_n}\}$= 1, in (X,T). This implies that 1 − $\wedge_{n \in N}$ (μ_{α_n}) = 1, and thus $\wedge_{n \in N}$ $\{\mu_{\alpha_n}\}$ = 0, in (X,T).

Proposition 3.3: If ($\{\lambda_\alpha\}_{\alpha \in \Delta}$)'s are fuzzy pseudo open sets in a fuzzy pseudo Lindelof space (X,T), such that, $\vee_{\alpha \in \Delta}$ $\{\lambda_\alpha\}$ = 1, then there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $\wedge_{n \in N}\{1- \lambda_{\alpha_n}\}$= 0, where cl$\{1- \lambda_{\alpha_n}\} \neq 1$ in (X,T).

Proof: Let (X,T) be a fuzzy pseudo Lindelof space. Then, for the cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of X by fuzzy pseudo open sets in (X,T), there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ of X by fuzzy pseudo open sets. That is, there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $\vee_{n \in N}\{\lambda_{\alpha_n}\}$= 1and hence $\wedge_{n \in N}\{1-\lambda_{\alpha_n}\}$= 0 in (X,T). Since $\{\lambda_{\alpha_n}\}$'s are fuzzy pseudo open sets in (X,T), $\lambda_{\alpha_n} = \mu_{\alpha_n} \vee \delta_{\alpha_n}$, where $\{\mu_{\alpha_n}\}$'s are non zero fuzzy pseudo open sets in and $\{\delta_{\alpha_n}\}$'s are fuzzy first category sets in (X,T). Now, int(λ_{α_n}) = int $(\mu_{\alpha_n} \vee \delta_{\alpha_n}) \geq$ int $(\mu_{\alpha_n}) \vee$ int $(\delta_{\alpha_n}) = \mu_{\alpha_n} \vee$ int $(\delta_{\alpha_n}) \neq 0$. Hence int$(\lambda_{\alpha_n}) \neq 0$. Then, cl$\{1- \lambda_{\alpha_n}\}$= 1- int$(\lambda_{\alpha_n}) \neq 1$, in (X,T).Thus $\wedge_{n \in N}$ $\{1- \lambda_{\alpha_n}\}$= 0, where cl$\{1- \lambda_{\alpha_n}\}$ $\neq$ 1in (X,T).

Proposition 3.4: If (X,T) is a fuzzy pseudo Lindelof space, then $\wedge_{n \in N}(\mu_{\alpha_n} \wedge \delta_{\alpha_n})$ = 0, where$\{\mu_{\alpha_n}\}$'s are fuzzy closed sets and $\{\delta_{\alpha_n}\}$'s are fuzzy residual sets in (X,T) .

Proof: Let (X,T) be a fuzzy pseudo Lindelof space. Then, for every cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of X by fuzzy pseudo open sets, there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ of X by fuzzy pseudo open sets. That is, $\vee_{\alpha \in \Delta} \{\lambda_\alpha\} = 1$ implies that $\vee_{n \in N} \{\lambda_{\alpha_n}\} = 1$, where $\{\lambda_{\alpha_n}\}$'s are fuzzy pseudo open sets in (X,T). Now $1 - \vee_{n \in N} \{\lambda_{\alpha_n}\} = 0$, implies that $\wedge_{n \in N} (1 - \lambda_{\alpha_n}) = 0$. By the theorem 2.2, $1 - \lambda_{\alpha_n} = (\mu_{\alpha_n} \wedge \delta_{\alpha_n})$, in (X,T), and this implies that $\wedge_{n \in N} (\mu_{\alpha_n} \wedge \delta_{\alpha_n}) = 0$, where $\{\mu_{\alpha_n}\}$'s are fuzzy closed sets and $\{\delta_{\alpha_n}\}$'s are fuzzy residual sets in (X,T).

Example 3.2: Let X = {a,b,c}.The fuzzy sets λ ,β, δ, λ, μ, γ, η and σ are defined on X as Follows:

α : X → [0,1] is defined as α(a) = 0.5 ; α(b) = 1; α(c)=0.4;
β : X → [0,1] is defined as β(a) = 1; β(b) = 0.6; β(c)=0.1;
δ : X → [0,1] is defined as δ(a) = 0.6; δ(b) =0.4; δ(c)=1;
λ : X → [0,1] is defined as λ(a) = 1 ; λ(b) =0.6; λ(c)=0.9;
μ : X → [0,1] is defined as μ(a) = 0.6; μ(b) = 0.6; μ(c)=0.9;
γ : X → [0,1] is defined as γ(a) = 0.5; γ(b) = 1; γ(c)=0.9;
η : X → [0,1] is defined as η(a) =0.6; η(b) = 1; η(c)=0.9;
σ : X → [0,1] is defined as σ(a) = 1; σ(b) = 0.6; σ(c)=0.4;

Then T = { 0, α, β, δ, α∨β, α∨δ, β∨δ, α∧β, α∧δ, β∧δ, α∨(β∧δ), β∨(α∧δ), δ∨(α∧β), α∧(β∨δ),β∧(α∨δ), δ∧(α∨β), α∧(β∧δ),1} is a fuzzy topology on X on computation, the fuzzy nowhere dense sets in (X,T), are – α,1– β,1– δ,1– α∨β,1– α∨δ, 1– β∨δ, 1– β∧δ,1– α∨(β∧δ), 1– β∨(α∧δ), 1– δ∨(α∧β), 1– α∧(β∨δ),1– β∧(α∨δ),1– δ∧(α∨β), 1– α∧(β∧δ),1–λ,1–γ,1–η, 1–μ and 1–σ and 1– α∧(β∧δ) is a fuzzy first category set in (X,T). The fuzzy pseudo open sets in (X, T) are 1 – [α∧(β∧δ)], α∨δ,β∨δ,[δ∨(α∧β)], μ, γ, θ, σ, and ω. Then α∨δ ∨ β∨δ ∨ [δ∨(α∧β)] ∨ μ ∨ λ ∨ γ ∨ σ ∨ η ≠ 1 and there is not a fuzzy pseudo Lindelöf space in (X,T).

Proposition 3.5: If (X,T) is a fuzzy pseudo Lindelof and fuzzy hyperconnected space, then (X,T) is a fuzzy simply Lindelof space.

Proof: Let $\{\lambda_\alpha\}_{\alpha \in \Delta}$ be a cover of X by fuzzy pseudo open sets in (X,T). Then, $\vee_{\alpha \in \Delta} \{\lambda_\alpha\} = 1$,in (X,T). Since (X,T) is a fuzzy hyperconnected space, by theorem 2.1, the fuzzy pseudo open sets $\{\lambda_\alpha\}$'s are fuzzy simply open sets in (X,T). Thus $\vee_{\alpha \in \Delta} \{\lambda_\alpha\} = 1$, implies that $\{\lambda_\alpha\}_{\alpha \in \Delta}$ be a cover of X by fuzzy simply open sets in (X,T). Since (X,T) is a fuzzy pseudo Lindelof space, the cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of X by fuzzy pseudo open sets has a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ of X. That is,$\vee_{n \in N}\{\lambda_{\alpha_n}\} = 1$ in (X,T). Also since (X,T) is a fuzzy hyperconnected space, the fuzzy pseudo open sets $(\{\lambda_{\alpha_n}\}_{n \in N})$'s are fuzzy simply open sets in (X,T). Thus the cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of X by fuzzy simply open sets has a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ implies that (X,T) is a fuzzy simply Lindelof space.

Proposition 3.6: If $\vee_{\alpha \in \Delta}\{\lambda_\alpha\} = 1$, where $\{\lambda_\alpha\}$'s are fuzzy simply open sets in a fuzzy hyperconnected and fuzzy pseudo Lindelof space, then $\vee_{n \in N}\{\lambda_{\alpha_n}\} = 1$, where $\{\lambda_{\alpha_n}\}$'s are fuzzy resolvable sets in (X,T).

Proof: Suppose that $\vee_{\alpha \in \Delta}\{\lambda_\alpha\} = 1$, where $\{\lambda_\alpha\}$'s are fuzzy simply open sets in (X,T). Since (X,T) is fuzzy pseudo Lindelof and fuzzy hyperconnected space, by proposition.3.2, (X,T) is a fuzzy simply Lindelof space and hence for the cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of X by fuzzy simply open sets, there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ of X. That is, $\vee_{n \in N}\{\lambda_{\alpha_n}\} = 1$ in (X,T). Also since the fuzzy simply open sets are fuzzy resolvable sets in (X,T), by theorem 2.3, $\vee_{n \in N}\{\lambda_{\alpha_n}\} = 1$, where $\{\lambda_{\alpha_n}\}$'s are fuzzy resolvable sets in (X,T).

Proposition 3.7: If (X,T) is a fuzzy pseudo Lindelof and fuzzy strongly irresolvable space, then (X,T) is a fuzzy simply Lindelof space.

Proof: Let $\{\lambda_\alpha\}_{\alpha \in \Delta}$ be a cover of X by fuzzy pseudo open sets in (X,T). Then, $\vee_{\alpha \in \Delta} \{\lambda_\alpha\} = 1$, in (X,T). Since (X,T) is a fuzzy strongly irresolvable space, by theorem 2.1, the fuzzy pseudo open sets $\{\lambda_\alpha\}$'s are fuzzy simply open sets in (X,T). Thus $\vee_{\alpha \in \Delta} \{\lambda_\alpha\} = 1$, implies that $\{\lambda_\alpha\}_{\alpha \in \Delta}$ be a cover of X by fuzzy simply open sets in (X,T). Since (X,T) is a fuzzy pseudo Lindelof space, the cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of X by fuzzy pseudo open sets has a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ of X. That is, $\vee_{n \in N}\{\lambda_{\alpha_n}\} = 1$ in (X,T). Also since (X,T) is a fuzzy strongly irresolvable space, the fuzzy pseudo open sets $\{\lambda_{\alpha_n}\}$'s are fuzzy simply open sets in (X,T). Thus the cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of X by fuzzy simply open sets has a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ implies that (X,T) is a fuzzy simply Lindelof space.

Proposition 3.8: If (X,T) is a fuzzy pseudo Lindelof and fuzzy strongly irresolvable space, then (X,T) is a fuzzy second category space.

Proof: Let (X,T) be a fuzzy pseudo Lindelof and fuzzy strongly irresolvable space. Then, by proposition 3.3, (X,T) is a fuzzy simply Lindelof space and hence by theorem 2.5, (X,T) is a fuzzy second category space.

Proposition 3.9: If $(\{\lambda_\alpha\}_{\alpha \in \Delta})$'s are fuzzy pseudo open sets such that $\vee_{\alpha \in \Delta}\{\lambda_\alpha\} = 1$, in a fuzzy pseudo Lindelof space (X,T) in which fuzzy first category sets are dense, then $\vee_{n \in N} \{\lambda_{\alpha_n}\} = 1$, where $cl\{\lambda_{\alpha_n}\} = 1$, in (X,T).

Proof: Let (X,T) be a fuzzy pseudo Lindelof space. Then for the cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of X by fuzzy pseudo open sets in (X,T), there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ of X by fuzzy pseudo open sets. That is, $\vee_{n \in N}\{\lambda_{\alpha_n}\} = 1$ in (X,T). Since $\{\lambda_{\alpha_n}\}$'s are fuzzy pseudo open sets in (X,T), $\lambda_{\alpha_n} = (\mu_{\alpha_n} \vee \delta_{\alpha_n})$, where $\{\mu_{\alpha_n}\}$'s are non zero fuzzy pseudo open sets and $\{\delta_{\alpha_n}\}$'s are fuzzy first category sets in (X,T). By hypothesis $cl\{\lambda_{\alpha_n}\} = 1$, in (X,T). Now, $cl(\lambda_{\alpha_n}) = cl(\mu_{\alpha_n} \vee \delta_{\alpha_n}) = cl(\mu_{\alpha_n}) \vee cl(\delta_{\alpha_n}) = \mu_{\alpha_n} \vee 1 = 1$, and hence $cl(\lambda_{\alpha_n}) = 1$. Thus $\vee_{n \in N} \{\lambda_{\alpha_n}\} = 1$, where $cl\{\lambda_{\alpha_n}\} = 1$, in (X,T).

Proposition 3.10 : If $\vee_{\alpha \in \Delta}\{\lambda_\alpha\} = 1$, where $\{\lambda_\alpha\}$'s are fuzzy pseudo open sets in a fuzzy hyperconnected and fuzzy pseudo Lindelof space, then $\vee_{n \in N} \{\lambda_{\alpha_n}\} = 1$, where $\{\lambda_{\alpha_n}\}$'s are fuzzy resolvable sets in (X,T).

Proof: suppose that $\vee_{\alpha \in \Delta}\{\lambda_\alpha\} = 1$, where $\{\lambda_\alpha\}$'s are fuzzy pseudo open sets in (X,T). Since (X,T) is a fuzzy pseudo Lindelof space, the cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of X by fuzzy pseudo open has a countable sub cover $\{\lambda_{\alpha_n}\}_{n \in N}$ of X by fuzzy pseudo open sets in (X,T). That is, $\vee_{n \in N}\{\lambda_{\alpha_n}\} = 1$ in (X,T). Also since (X,T) is a fuzzy hyperconnected space, the pseudo open sets $\{\lambda_{\alpha_n}\}$'s are fuzzy resolvable sets in (X,T).Thus $\vee_{n \in N} \{\lambda_{\alpha_n}\} = 1$, where $\{\lambda_{\alpha_n}\}$'s are fuzzy resolvable sets in (X,T).

Proposition 3.11: If $int(\vee_{\alpha \in \Delta}\{\lambda_\alpha\}) = 0$, where $\{\lambda_\alpha\}$'s are fuzzy pseudo Lindelof and fuzzy hyperconnected space (X,T), then (X,T) is a fuzzy Baire space.

Proof: suppose that $int(\vee_{\alpha \in \Delta} \{\lambda_\alpha\}) = 0$, where $\{\lambda_\alpha\}$'s are fuzzy pseudo Lindelof and fuzzy hyperconnected space (X,T). Then, by proposition.3.6, $\{\lambda_\alpha\}$'s are fuzzy resolvable sets in (X,T).By hypothesis, $int(\vee_{\alpha \in \Delta} \{\lambda_\alpha\}) = 0$, where $\{\lambda_\alpha\}$'s are fuzzy resolvable sets in (X,T).Then, by theorem 2.4, (X,T) is a fuzzy Baire space.

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