One Point Union Cordial Labeling of Graphs Related to **triple -Tail** of C_4 and invariance

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Abstract: **We discuss graphs of type $G^{(k)}$ i.e. one point union of k-copies of G for cordial labeling. We take G as triple-tail graph. A triple-tail graph is obtained by attaching a path P_m to any three vertices which forms a path p_3 in given graph C_4. It is denoted by triple- tail(G,P_m) where G is given graph and all the three tails are identical to p_m. We take G as C_4 and restrict our attention to m = 2, 3 and 4 in P_m. Further we consider all possible structures of $G^{(k)}$ by changing the common point and obtain non-isomorphic structures. We show all these structures as cordial graphs. This is called as invariance of different structures of $G^{(k)}$ under cordial labeling.**

Keywords: cordial, one point union, triple-tail graph, cycle, labeling, vertex.

Subject Classification: 05C78

1. Introduction

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Holton [6] , Graph Theory by Harary [7], A dynamic survey of graph labeling by J.Gallian [9] and Douglas West.[10].I.Cahit introduced the concept of cordial labeling[6]. f:V(G)→{0,1} be a function. From this label of any edge (uv) is given by |f(u)-f(v)|. Further number of vertices labeled with 0 i.e $v_f(0)$ and the number of vertices labeled with 1 i.e.$v_f(1)$ differ at most by one .Similarly number of edges labeled with 0 i.e.$e_f(0)$ and number of edges labeled with 1 i.e.$e_f(1)$ differ by at most one. Then the function f is called as cordial labeling. Cahit has shown that : every tree is cordial; Kn is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n is not congruent to 3 (mod 4).A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian [9].

Our focus of attention is on one point unions on C_4 graphs. For a given graph there are different one point unions (upto isomorphism) structures possible in $G^{(k)}$. It depends on which point on G is used to fuse to obtain one point union. It is called as invariance under cordial labeling. We use the convention that $v_f(0,1)$ = (a,b) to indicate the number of vertices labeled with 0 are a in number and that number of vertices labeled with 1 are b. Further $e_f(0,1)$ = (x,y) we mean the number of edges labeled with o are x and number of edges labeled with 1 arey. The graph whose cordial labeling is available is called as cordial graph. In this paper we define triple -tail graph and obtain one point union graphs on it. For this we consider C_4 and t-pendent edges attached to each of any three vertices of C_4.($t\leq 4$)

2. Preliminaries

3.1 Tail Graph: A (p,q) graph G to which a path P_m is fused at some vertex. This also can be explained as take a copy of graph G and at any vertex of it fuse a path P_m with it's one of the pendent vertex. It's number of vertices are P+m-1 and edges are by q + m-1. It is denoted by tail(G, P_m).

3.2 double-tail graph of G is denoted by double-tail(G,Pm).It is obtained by attaching (fusing) path P_m to a pair of adjacent vertices of G .It has q+2m-2 edges and p + 2m-2 vertices.($m\geq 2$)

3.3 Fusion of vertices. Let $u \neq v$ be any two vertices of G. We replace these two vertices by a single vertex say x and all edges incident to u and v are now incident to x. If loop is formed then it is deleted.[6]

3.4 $G^{(K)}$ it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies. If G is a (p, q) graph then $|V(G_{(k)}|$ = k(p-1)+1and |E(G)|= k.q

3.4 triple-tail graph of G is denoted by triple-tail(G,P_m).It is obtained by attaching (fusing) path Pm to each of three vertices of G that forms a path P_3 .It has q+3m-3 edges and p + 3m-3 vertices.($m\geq 2$)

Results Proved:

Theorem4.1 All non- isomorphic one point union on k-copies of graph obtained on G = triple –tail(C_4,p_2) given by $G^{(k)}$ are cordial graphs. Proof: From fig.4.1 it follows that there are five non-isomorphic structures of one point union possible at vertices a, b, c.

Define f:V(G)→{0,1} that gives us labeled copies of G as given below..We extend the same f : V($G^{(k)}$):→{0,1} to obtain cordial labeling of $G^{(k)}$. When the one point union is taken at point a then type A and type B label are fused alternately at vertex a.

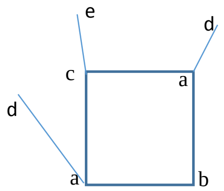

Fig. 4.1 One Point Union may be taken at vertices ‘a’, ‘b’,‘c’,’d’,’e’

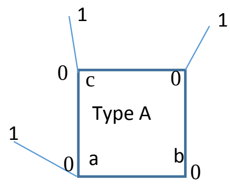

Fig. 4.2 $v_f(0,1)=(4,3)$; $e_f(0,1) = (4,3)$

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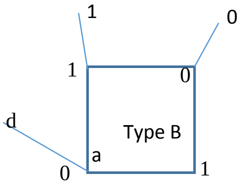

Fig. 4.3 $v_f(0,1)=(4,3)$; $e_f(0,1) = (3,4)$

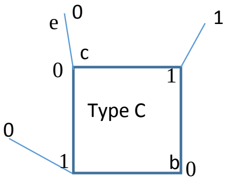

Fig. 4.4 $v_f(0,1)=(4,3)$; $e_f(0,1) = (3,4)$

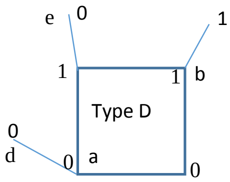

Fig. 4.5 $v_f(0,1)=(4,3)$; $e_f(0,1) = (4,3)$

To obtain one point union of k copies of G at vertex a, when k =1 we use type A label. For k>1 fuse type A and type B label at vertex a.When k = 2x there will be x copies of type A and type B each.When k = 2x+1 there will be x+1 copies of typea label and x copies of type B label. The label number distribution is $v_f(0,1)$ =(4+6x,3+6x), $e_f(0,1)$= (4+ 7x,3+7x).when k = 2x+1, x= 0,1, 2, .. The label number distribution is $v_f(0,1)$ =(7+6(x-1),6+6(x-1)), $e_f(0,1)$= (7+ 7(x-1),7+7(x-1)).when k = 2x, x= 1, 2, ..

To obtain one point union of k copies of G at vertex c or b , when k =1 we use type A label. For k>1 fuse type A and type C label at vertex c (or b).When k = 2x there will be x copies of type A and type B each. When k = 2x+1 there will be x+1 copies of type A label and x copies of type C label. The label number distribution is $v_f(0,1)$ =(4+6x,3+6x), $e_f(0,1)$= (4+ 7x,3+7x).when k = 2x+1, x= 0,1, 2, .. The label number distribution is $v_f(0,1)$ =(7+6(x-1),6+6(x-1)), $e_f(0,1)$= (7+ 7(x-1),7+7(x-1)),when k = 2x, x= 1, 2, .. To obtain one point union of k copies of G at vertex d, when k =1 we use type D label. For k>1 fuse type D and type B label at vertex d. When k = 2x there will be x copies of type D and type B each. When k = 2x+1 there will be x+1 copies of type D label and x copies of type B label. The label number distribution is $v_f(0,1)$ =(4+6x,3+6x), $e_f(0,1)$= (4+ 7x,3+7x).when k = 2x+1, x= 0,1, 2, .. The label number distribution is $v_f(0,1)$ =(7+6(x-1),6+6(x-1)), $e_f(0,1)$= (7+ 7(x-1),7+7(x-1)).when k = 2x, x= 1, 2, ..

To obtain one point union of k copies of G at vertex e, when k =1 we use type D label. For k>1 fuse type D and type C label at vertex e. When k = 2x there will be x copies of type D and type C each. When k = 2x+1 there will be x+1 copies of type D label and x copies of type C label. The label number distribution is $v_f(0,1)$ =(4+6x,3+6x), $e_f(0,1)$= (4+ 7x,3+7x).when k = 2x+1, x= 0,1, 2, .. The label number distribution is $v_f(0,1)$ =(7+6(x-1),6+6(x-1)), $e_f(0,1)$= (7+ 7(x-1),7+7(x-1)).when k = 2x, x= 1, 2, .. Thus the different structures obtained on $G^{(k)}$ are cordial.

Theorem 4.2 All non- isomorphic one point union on k-copies of graph obtained on G =triple-tail(C_4,P_3) given by $G^{(k)}$ are cordial graphs.

Proof: From fig 4.6 it follows that there are 5 non-isomorphic structure at points a, b, c, d, e possible at which can be obtained on one point union of k copies of graph.

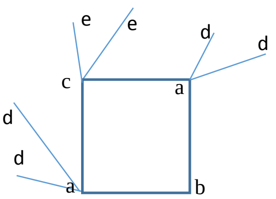

Fig. 4.6 One Point Union may be taken at vertices ‘a’, ‘b’,‘c’,’d’,’e’

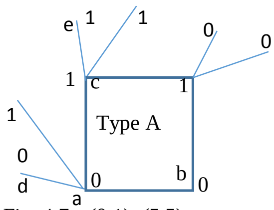

Fig. 4.7 $v_f(0,1)=(5,5)$; $e_f(0,1) = (5,5)$

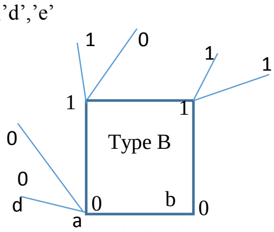

Fig. 4.7 $v_f(0,1)=(6,4)$; $e_f(0,1) = (5,5)$

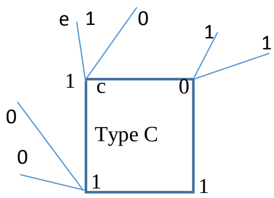

Fig. 4.8 $v_f(0,1)=(4,6)$; $e_f(0,1) = (5,5)$

Define f:V(G)→{0,1} that gives us labeled copies of G as above. We extend the same f : V($G^{(k)}$):→{0,1} to obtain cordial labeling of $G^{(k)}$. To obtain one point union at points a or b or d we fuse type A label with type B label at one of these required points. When k = 2x type A and type B are used x times each. When k = 2x+1 then type A label is used x+1 times and type B label for x times to obtain $G^{(K)}$.The label distribution is $v_f(0,1)$ =(5+9x,5+9x), $e_f(0,1)$= (5k,5k).when k = 2x+1, x= 0,1, 2, .. The label number distribution is $v_f(0,1)$ =(10+9(x-1),9+9(x-1)), $e_f(0,1)$= (5k,5k)).when k = 2x, x= 1, 2,

To obtain one point union at points e or c we fuse type A label with type C label at one of these required points. When k = 2x type A and type C are used x times each. When k = 2x+1 then type A label is used x+1 times and type C label for x times to obtain $G^{(K)}$.The label distribution is $v_f(0,1)$ =(5+9x,5+9x), $e_f(0,1)$= (5k,5k).when k = 2x+1, x= 0,1, 2, .. The label number distribution is $v_f(0,1)$ =(9+9(x-1),10+9(x-1)), $e_f(0,1)$= (5k,5k)).when k = 2x, x= 1, 2, ..Thus the graph is cordialand invariance under cordiality is observed.

Theorem 4.3 All non- isomorphic one point union on k-copies of graph obtained on G = triple- tail(C_4,P_2) given by $G^{(k)}$ are cordial graphs.(except possibly when one point union is taken at degree two vertex of cycle C_4.)

Proof: From figure 4.9 it follows that one can take one point union at vertices a, b, c, d, e. a

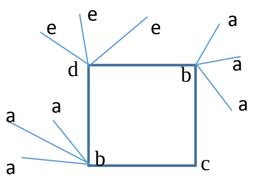

Fig. 4.9 One Point Union may be taken at vertices 'a', 'b' 'c', 'd','e'.

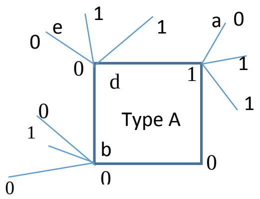

Fig. 4.11 $v_f(0,1)$=(7,6) ; $e_f(0,1)$ = (7,6)

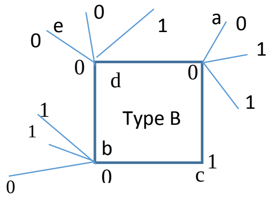

Fig. 4.11 $v_f(0,1)$=(7,6) ; $e_f(0,1)$ = (6,7)

Define f:V(G)→{0,1} that gives us labeled copies of G as above.We extend the same f : V($G^{(k)}$):→{0,1} to obtain cordial labeling of $G^{(k)}$. To achieve this we fuse type A label with type B label at point a (at point b) (at point d)(at point e).These two types of labels are used alternately. The label number distribution is $v_f(0,1)$ = (6k+1,6k) and $e_f(0,1)$ = (7+13x , 6+13x) when k is odd number given by 2x+1, x=0, 1,2,..And when k is an odd number given by k = 2x,x=1, .. we have $v_f(0,1)$ = (6k+1,6k) and $e_f(0,1)$=(13k,13k) .Note that the common point to all copies is vertex with label '0'.At vertex c the one point union on k copies of G with cordial label can be obtained only at few stray cases. Eccept for the point c the graph is cordial.

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Theorem 4.4 All non- isomorphic one point union on k-copies of graph obtained on G = triple- tail(C_4,3P_2) given by $G^{(k)}$ are cordial graphs.

Proof: from fig 4.12 it is clear that we can take one point union at five different vertices a, b, c, d, and e.

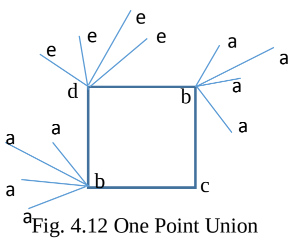
Fig. 4.12 One Point Union may be taken at vertices 'a', 'b' 'c','d','e'

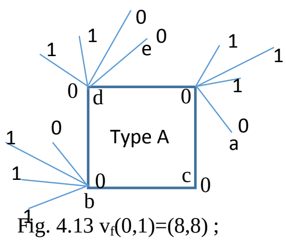

Fig. 4.13 $v_f(0,1)$=(8,8) ; $e_f(0,1)$ = (8,8)

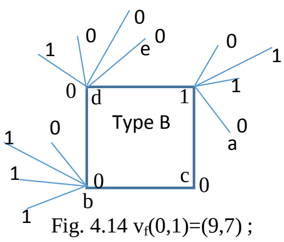

Fig. 4.14 $v_f(0,1)$=(9,7) ; $e_f(0,1)$ = (8,8)

Define f:V(G)→{0,1} that gives us labeled copies of G as above.We extend the same f : V($G^{(k)}$):→{0,1} to obtain cordial labeling of $G^{(k)}$. To achieve this we fuse type A label with type B label at point a (at point b) (at point d)(at point e),(at point c).These two types of labels are used alternately. When k = 2x the type A label and type B label will each appear for x times. When k = 2x+1 type A label will appear for x+1 times and type B label will appear for x times. The label number distribution is $v_f(0,1)$ = (8+8(x),8+8(x)) and $e_f(0,1)$ = (8k,8k) when k is odd number given by 2x+1, x=0, 1, 2,..When k is an even number given by k = 2x,x=1, 2, .. we have $v_f(0,1)$ = (16+15(x-1),15+15(x-1)) and $e_f(0,1)$=(8k,8k) .Note that the common point to all copies is vertex with label '0'. Thus the graph is cordial. Conclusions: In this paper we define some new families obtained from C_4 .We take a copy of C_4 and to any three of it's vertices fuse t pendent edges each. We call this as triple-tail (G,tP_2) graph.. We show that

1) All non- isomorphic one point union on k-copies of graph obtained on G = triple-tail(C_4,P_2) given by $G^{(k)}$ are cordial graphs.

2) All non- isomorphic one point union on k-copies of graph obtained on G = triplele-tail(C_4,$2P_2$) given by $G^{(k)}$ are cordial graphs.

3) All non- isomorphic one point union on k-copies of graph obtained on G =triplele-tail(C_4,$3P_3$) given by $G^{(k)}$ are cordial graphs. (except possibly at degree two vertex)

4) All non- isomorphic one point union on k-copies of graph obtained on G = triple-tail(C_4,$4P_2$) given by $G^{(k)}$ are cordial graphs. It is necessary to investigate the cordiality and invariance for for one point union graph for the general case when t pendent edges are attached at each three vertices of C_4.

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