Cordiality of Mixed Graphs' One Point Union

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***Abstract*: Instead of taking one point union on a graph G alone we take it on two graphs G_1 and G_2.This structure is denoted by $G = (G_1,G_2)^{(k)}$. Care is taken that G_1 and G_2 are repeated in G in a certain sequence. If both G_1 and G_2 appear alternately then the graph G is balanced graph and for given k the number of copies of G_1 and that of G_2 differ at most by 1 in G. We discuss cordiality of G by taking G_1 and G_2 from C3, C_4, C_5 and house graph.**

***Keywords*: mixed graphs, cordial labeling, cycle graph, union, structure.**

Subject Classification: 05C78

Introduction:

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [6], A dynamic survey of graph labeling by J.Gallian [8] and Douglas West.[9].I.Cahit introduced the concept of cordial labeling[5]. $f:V(G)\rightarrow\{0,1\}$ be a function. From this label of any edge (uv) is given by $|f(u)-f(v)|$. Further number of vertices labeled with 0 i.e $v_f(0)$ and the number of vertices labeled with 1 i.e.$v_f(1)$ differ at most by one .Similarly number of edges labeled with 0 i.e. $e_f(0)$ and number of edges labeled with 1 i.e.$e_f(1)$ differ by at most one. Then the function f is called as cordial labeling. Cahit has shown that : every tree is cordial; K_n is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n is not congruent to 3 (mod 4).A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian [8]. Let G_1 and G_2 be any two graphs. $G^{(k)}$ is one point union of k copies of G in which a fixed vertex x of G is chosen and K copies of G are fused together at x. By $G = (G_1,G_2)^{(k)}$ we have taken one point union of G_1 and G_2 and not G alone. For that a fixed vertex from G_1 say p and a fixed vertex from G_2 say q is chosen at which m copies of G_1 and n copies of G_2 are fused together to obtain $G = (G_1,G_2)^{(k)}$.Note that m+n =k. If both G_1 and G_2 appear alternately then the graph G is balanced graph and for given k the number of copies of G_1 and that of G_2 differ at most by 1 in G. i.e. $|m-n|\leq 1$. We have taken G_1 and G_2 from C_3, C_4, house graph and C_5.

4. Theorems proved:

4.1 Theorem. Mixed one point union of C_3 and C_5 given by $G^{(k)}= (C_3, C5)^{(k)}$ is cordial.

Proof: Define $f:V(G)\rightarrow\{0,1\}$ as follows. f gives different types of labeled copies of C_3 and C_5 as given below.

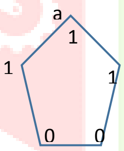

Fig 4.1 labeled copy of C_5
$v_f(0,1) =(2,3)$, $e_f(0,1) = (3,2)$

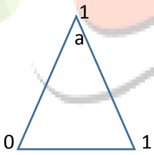

Fig 4.2 labeled copy of C_3
$v_f(0,1) =(1,2)$, $e_f(0,1) = (1,2)$

In construction of labeled copy of $G^{(k)}= (C_3,C_5)^{(k)}$ we use i^{th} copy as labeled copy of C_5 as in fig.4.1 when $i\equiv1$(mod2) and when $i \equiv 0$(mod2) we use labeled copy of C_3 as in fig 4.2. Both copies are fused at vertex 'a' on it. The label numbers are as follows: On vertices we have $v_f(0,1) =(2+3x,3+3x)$, and on edges :$e_f(0,1) = (4x+3,4x+2)$ for k is of type 2x+1 x= 0,1,2,…If k =2x, x=1, 2, 3.. $v_f(0,1) =(3x,1+3x)$, and $e_f(0,1) = (4x,4x)$. $G^{(k)}$ is balanced graph as alternate copies are C_5 and C_3 in labeled copy of G.

4.2 Theorem: Mixed one point union of C_3 and house graph given by $G^{(k)}= (C_3, \text{house})^{(k)}$ is cordial.

Proof: Define $f:V(G)\rightarrow\{0,1\}$ as follows. f gives different types of labeled copies of C_3 and house as given below. In construction of $G^{(k)}$, when k =1 Type A copy of house is used. For k =2, labeled copy in fig 4.5 is used. For all rest of i when $i \geq 3$, the i^{th} copy in $G^{(k)}$ is Type A label if $i \equiv 3$(mod 4), Type B label if $i\equiv 0$ (mod 4), labeled copy of C_3 if $i\equiv 1, 2$ (mod 4) . The label numbers in resultant graph are as follows: On vertices we have $v_f(0,1) =(2,3)$, and on edges :$e_f(0,1) = (3, 3)$ for k= 1, for k= 2 we have $v_f(0,1) =(3,4)$, and on edges : $e_f(0,1) = (4,5)$,for all $k\geq3$ we have:

i) if k is of type 4x+3, x= 0,1,2,… .. $v_f(0,1) =(5+6x,6+6x)$, and $e_f(0,1) = (7+9x,8+9x)$.

ii) If k =4x, x=1, 2, 3.. .. $v_f(0,1) =(7+6(x-10,8+6(x-1))$, and $e_f(0,1) = (11+9(x-1),10+9(x-1))$.

iii) if k is of type 4x+1,x= 0,1,2.. $v_f(0,1) =(8+6x,9+6x)$, and $e_f(0,1) = (12+9x,12+9x)$.

iv) if k is of type 4x+2, x= 0,1,2.. $v_f(0,1) = (9+6x,10+6x)$, and $e_f(0,1) = (13+9x,124+9x)$.

Note that $G^{(k)}$ is not balanced graph.

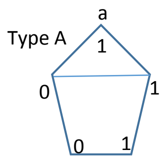

Fig 4.3 labeled copy of house: $v_f(0,1)$ =(2,3), $e_f(0,1)$ = (3,3)

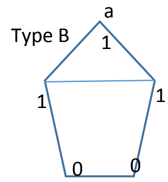

Fig 4.4 labeled copy of house: $v_f(0,1)$ =(2,3), $e_f(0,1)$ = (4,2)

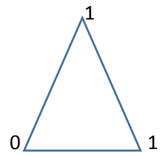

Fig 4.5 labeled copy of C_3 $v_f(0,1)$ =(1,2), $e_f(0,1)$ = (1,2)

Thus G is cordial graph. #

4.3 Theorem: Mixed one point union of C_3 and house graph given by $G^{(k)}= (C_5, C_6)^{(k)}$ is cordial. Proof: Define f:V(G)→{0,1} as follows. f gives different types of labeled copies of C_5 and C_6 as given below.

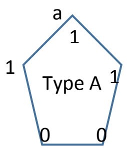

Fig 4.6 labeled copy of C_5 $v_f(0,1)$ =(2,3), $e_f(0,1)$ = (3,2)

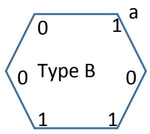

Fig 4.7 labeled copy of C_6 $v_f(0,1)$ =(3,3), $e_f(0,1)$ = (2,4)

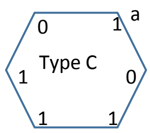

Fig 4.8 labeled copy of C_6 $v_f(0,1)$ =(3,3), $e_f(0,1)$ = (2,4)

To construct one point union two or more copies are fused at point 'a' on it.

When i) k=1 we have Type A to be used.

ii) When k =2 type B is to be used and label numbers are $v_f(0,1)$ =(5,5), $e_f(0,1)$ = (5,6).
iii) If k ≡ 3 (mod 6) Type A is used. Write k = 6x+3, x =0, 1, 2, , ,.. we have label distribution given by $v_f(0,1)$ = (13x+7,13x+7), $e_f(0,1)$ = (16x+8,16x+8).

iv) If k ≡ 4 (mod 6) Type A is used. Write k = 6x+4, x =0, 1, 2, , ,.. we have label distribution given by $v_f(0,1)$ = (13x+9,13x+9), $e_f(0,1)$ = (16x+11,16x+10).

v) If k ≡ 5 (mod 6) Type B is used. Write k = 6x+5, x =0, 1, 2, , ,.. we have label distribution given by $v_f(0,1)$ = (13x+12,13x+11), $e_f(0,1)$ = (16x+13,16x+14).

vi) If k ≡ 0 (mod 6) Type A is used. Write k = 6x, x =1, 2, , ,.. we have label distribution given by $v_f(0,1)$ = (13(x-1)+14,13(x-1)+13), $e_f(0,1)$ = (16(x-1)+16,16(-1)x+16).

vii) If k ≡ 1 (mod 6) Type A is used. Write k = 6x+1, x =1, 2, , ,.. we have label distribution given by $v_f(0,1)$ = (13(x-1)+16,13(x-1)+15), $e_f(0,1)$ = (16(x-1)+19,16(x-1)+18).

viii) If k ≡ 2 (mod 6) Type C is used. Write k = 6x+2, x =1, 2, , ,.. we have label distribution given by $v_f(0,1)$ = (13(x-1)+18,13(x-1)+18), $e_f(0,1)$ = (16(x-1)+21,16(x-1)+22).Thus the mixed graphs one point union is cordial.

4.4 Theorem: Mixed one point union of C_5 and house graph given by $G^{(k)}= (C_5, \text{house})^{(k)}$ is cordial. Proof: Define f:V(G)→{0,1} as follows. f gives different types of labeled copies of C_3 and house as given below. To construct one point union two or more two or more copies are fused at point 'a' on it. In construction of $G^{(k)}$, when k≡ 0 (mod 4) then k =4x, x= 1,2.. Fuse Type B with Type C at point 'a' on it. We have,

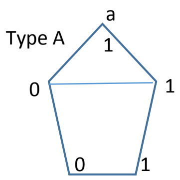

Fig 4.9 labeled copy of house: $v_f(0,1)$ =(2,3), $e_f(0,1)$ = (3,3)

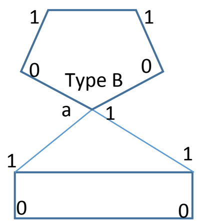

Fig 4.10 labeled copy of house: $v_f(0,1)$ =(4,5), $e_f(0,1)$ = (5,6)

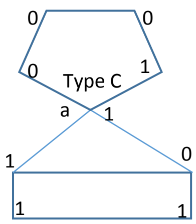

Fig 4.11 labeled copy of house: $v_f(0,1)$ =(4,5), $e_f(0,1)$ = (6,5)

label numbers given by $v_f(0,1)$ =(8x,8x+1), $e_f(0,1)$ = (11x,11x).

When k≡ 2 (mod 4) then k =4x +2. First obtain labeled copy for k = 4x and fuse a copy of type B at vertex 'a' on it with vertex 'a' on $G^{(4x)}$. We have label numbers given by $v_f(0,1)$ =(8x+4,8x+5), $e_f(0,1)$ = (11x+5,11x+6). When k≡ 1 (mod 4) then k =4x +1, x = 0,1,2… First obtain labeled copy of $G^{(4x)}$ and fuse it at vertex 'a' on it with Type A label at vertex 'a' on it. We have label numbers given by $v_f(0,1)$ =(8x+2,8x+3), $e_f(0,1)$ = (11x+3,11x+3). When k≡ 3 (mod 4) then k =4x +3. First obtain labeled copy of $G^{(4x+2)}$ and fuse it at vertex 'a' on it with Type A label at vertex 'a' on it. We have label numbers given by $v_f(0,1)$ =(8x+6,8x+7), $e_f(0,1)$ = (11x+8,11x+9).Thus $G^{(k)}$ is balanced and cordial.

4.5 Theorem: Mixed one point union of C_6 and C_3 graph given by $G^{(k)}= (C_6, C_3)^{(k)}$ is cordial.

Proof: Define f:V(G)→{0,1} as follows. f gives different types of labeled copies of C_3 and C_6 as given below.

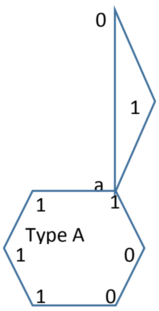

Fig 4.12 labeled copy $v_f(0,1)$ =(3,5), $e_f(0,1)$ = (5,4)

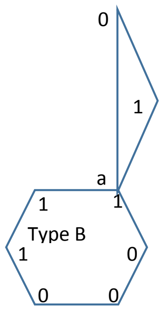

Fig 4.13 labeled copy $v_f(0,1)$ = (4,4), $e_f(0,1)$ = (5,4)

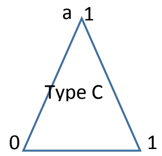

Fig 4.14 labeled copy of C_3 $v_f(0,1)$ =(1,2), $e_f(0,1)$ = (1,2)

In design of $G^{(k)}$ When we first obtain a block of $G^{(6)}$.Fuse Type A, Type B and two copies of Type C at vertex 'a' on it to obtain $G^{(6)}$ .We use this block repeatedly to obtain $G^{(6x)}$. At this stage we have label numbers given by $v_f(0,1)$ =(9x,9x+1), $e_f(0,1)$ = (12x,12x). where k= 6x, x=1, 2, ..

1)If k=1+6x, x= 0,1, .. First obtain labeled block on 6x copies as above. Fuse it with a copy of type C label at vertex 'a' on it. We have label numbers given by $v_f(0,1)$ =(9x+1,9x+2), $e_f(0,1)$ = (12x+1,12x+2). 2) k= 6x+2, x= 0,1, .. First obtain labeled block on 6x copies as above. Fuse it with a copy of type B label at vertex 'a' on it. We have label numbers given by $v_f(0,1)$ =(9x+4,9x+4), $e_f(0,1)$ = (12x+5,12x+4). 3) k= 6x+3, x= 0,1, .. First obtain labeled block on 6x copies as above. Fuse it with a copy of type A label and type C label at vertex 'a' on it. We have label numbers given by $v_f(0,1)$ =(9x+5,9x+5), $e_f(0,1)$ = (12x+6,12x+6). 4) k= 6x+4, x= 0,1, .. First obtain labeled block on 6x copies as above. Fuse it with a copy of type A label and two copies of type C label at vertex 'a' on it. We have label numbers given by $v_f(0,1)$ =(9x+6,9x+6), $e_f(0,1)$ = (12x+7,12x+8). 5) k= 6x+5, x= 0,1, .. First obtain labeled block on 6x copies as above. Fuse it with a copy of type A label, one copy of type B label and a copy of C label at vertex 'a' on it. We have label numbers given by $v_f(0,1)$ =(9x+8,9x+9), $e_f(0,1)$ = (12x+11,12x+10). This is not balanced graph. It is cordial graph.

4.6 Theorem: Mixed one point union of C_6 and house graph given by $G^{(k)}= (C_{6II}, \text{house})^{(k)}$ is cordial. Proof: : Define f:V(G)→{0,1} as follows. f gives different types of labeled copies of C_6 and house as given below.

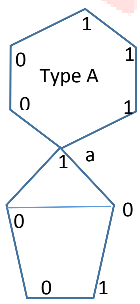

Fig 4.15 labeled copy of house: $v_f(0,1)$ =(5,5), $e_f(0,1)$ = (6,6)

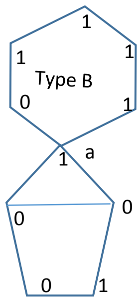

Fig 4.16 labeled copy of house: $v_f(0,1)$ =(4,6), $e_f(0,1)$ = (6,6)

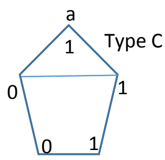

Fig 4.17 labeled copy of house: $v_f(0,1)$ =(2,3), $e_f(0,1)$ = (3,3)

In designing a labeled copy of $G^{(k)}$, Different label types are fused at point 'a' on it.When k =1 we use Type C label. $G^{(4x)}$ is obtained by fusing x copies of Type A and x copies of Type B at vertex 'a'. At this stage the label number distribution is $v_f(0,1)$ =(9x,9x+1), $e_f(0,1)$ = (12x,12x). Where k= 6x, x=1, 2, .. When k = 1+4x, x= 0,1, ..first obtain labeled block of $G^{(4x)}$.To this fuse type C at vertex 'a'. At this stage the label number distribution is $v_f(0,1)$ =(9x+2,9x+3), $e_f(0,1)$ = (12x+3,12x+3). When k = 4x+2, x= 0,1, ..first obtain labeled block of $G^{(4x)}$.To this fuse type A at vertex 'a'. At this stage the label number distribution is $v_f(0,1)$ =(9x+5,9x+5), $e_f(0,1)$ = (12x+6,12x+6). When k = 4x+3, x= 0,1, ..first obtain labeled block of $G^{(4x+3)}$ as above .To this fuse type C at vertex 'a'. At this stage the label number distribution is $v_f(0,1)$ =(9x+2,9x+2), $e_f(0,1)$ = (12x+9,12x+9).

The graph is balanced and cordial. \

Conclusions. We have discussed mixed one point union on C_3,C_5,C_6 and house graph by taking two of them together for cordial labeling. It is necessary to investigate the cordial labeling for taking three or more copies together.

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